POLITECNICO DI TORINO

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Graphs: Cycles

Tecniche di Programmazione – A.A. 2012/2013

Summary

- Definitions
- Algorithms

Definitions

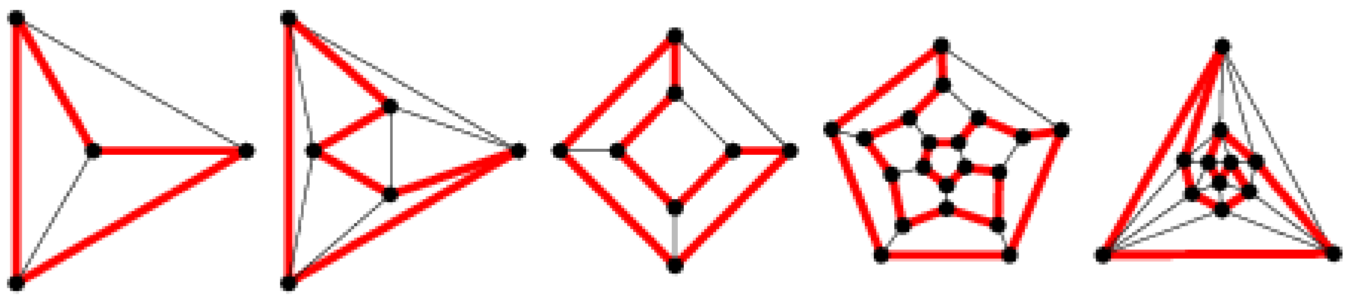
Graphs: Cycles

Cycle

- A **cycle** of a graph, sometimes also called a circuit, is a subset of the edge set of that forms a path such that the first node of the path corresponds to the last.

Hamiltonian cycle

- A cycle that uses each graph vertex of a graph exactly once is called a Hamiltonian cycle.



Hamiltonian path

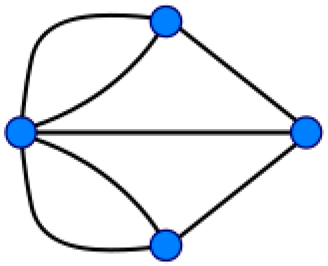
- A Hamiltonian path, also called a Hamilton path, is a path between two vertices of a graph that visits each vertex exactly once.
  - N.B. does not need to return to the starting point

Eulerian Path and Cycle

- An **Eulerian path**, also called an Euler chain, Euler trail, Euler walk, or "Eulerian" version of any of these variants, is a walk on the graph edges of a graph which **uses each graph edge** in the original graph **exactly once**.
- An **Eulerian cycle**, also called an Eulerian circuit, Euler circuit, Eulerian tour, or Euler tour, is a trail which starts and ends at the **same** graph vertex.

Theorem

- A connected graph has an Eulerian **cycle** if and only if it **all vertices have even degree**.
- A connected graph has an Eulerian **path** if and only if it has **at most two graph vertices of odd degree**.
  - …easy to check!



Königsberg Bridges

Weighted vs. Unweighted

- Classical versions defined on Unweighted graphs
- Unweighted:
  - Does such a cycle exist?
  - If yes, find at least one
    - Optionally, find all of them
- Weighted
  - Does such a cycle exist?
    - Often, the graph is complete ☺
  - If yes, find at least one
  - If yes, find **the best one** (with **minimum** weight)

Algorithms

Graphs: Cycles

Eulerian cycles: Hierholzer's algorithm (1)

- Choose **any** starting vertex *v*, and **follow a trail** of edges from that vertex until returning to *v*.
  - It is **not** possible to get stuck at any vertex other than *v*, because the even degree of all vertices ensures that, when the trail enters another vertex *w* there must be an unused edge leaving *w*.
  - The tour formed in this way is a **closed** tour, but may **not** cover all the vertices and edges of the initial graph.

Eulerian cycles: Hierholzer's algorithm (2)

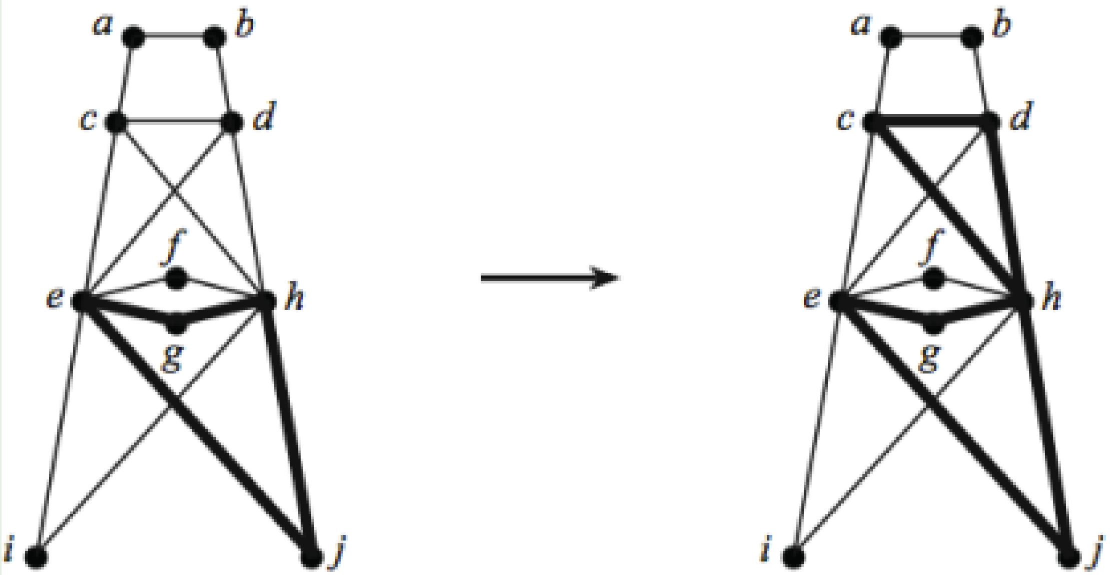
- As long as there exists a vertex *v* that belongs to the current tour but that has adjacent edges not part of the tour, **start another trail** from *v*, following **unused** edges until returning to *v*, **and join** the tour formed in this way to the previous tour.

Given: an Eulerian graph G

Find an Eulerian circuit of G.

1. Identify a circuit in G and call it R_1. Mark the edges of R_1. Let $i = 1$.
2. If R_i contains all edges of G, then stop (since R_i is an Eulerian circuit).
3. If R_i does not contain all edges of G, then let v_i be a node on R_i that is incident with an unmarked edge, e_i.
4. Build a circuit, Q_i, starting at node v_i and using edge e_i. Mark the edges of Q_i.
5. Create a new circuit, R_{i+1}, by patching the circuit Q_i into R_i at v_i.
6. Increment i by 1, and go to step (2).

Example

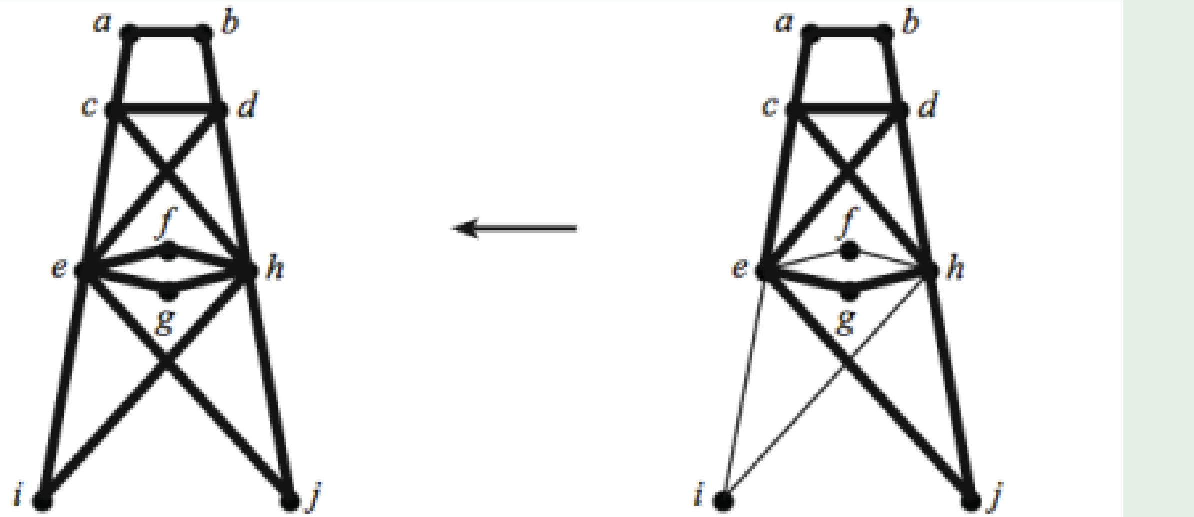


R_1: e, g, h, j, e $\quad$ R_2: $e, g, \mathbf{h}, \mathbf{d}, \mathbf{c}, \mathbf{h}, j, e$

Q_1: h, d, c, h $\quad$ Q_2: d, b, a, c, e, d

Finding Eulerian circuits
Hierholzer's Algorithm

Example (continued)

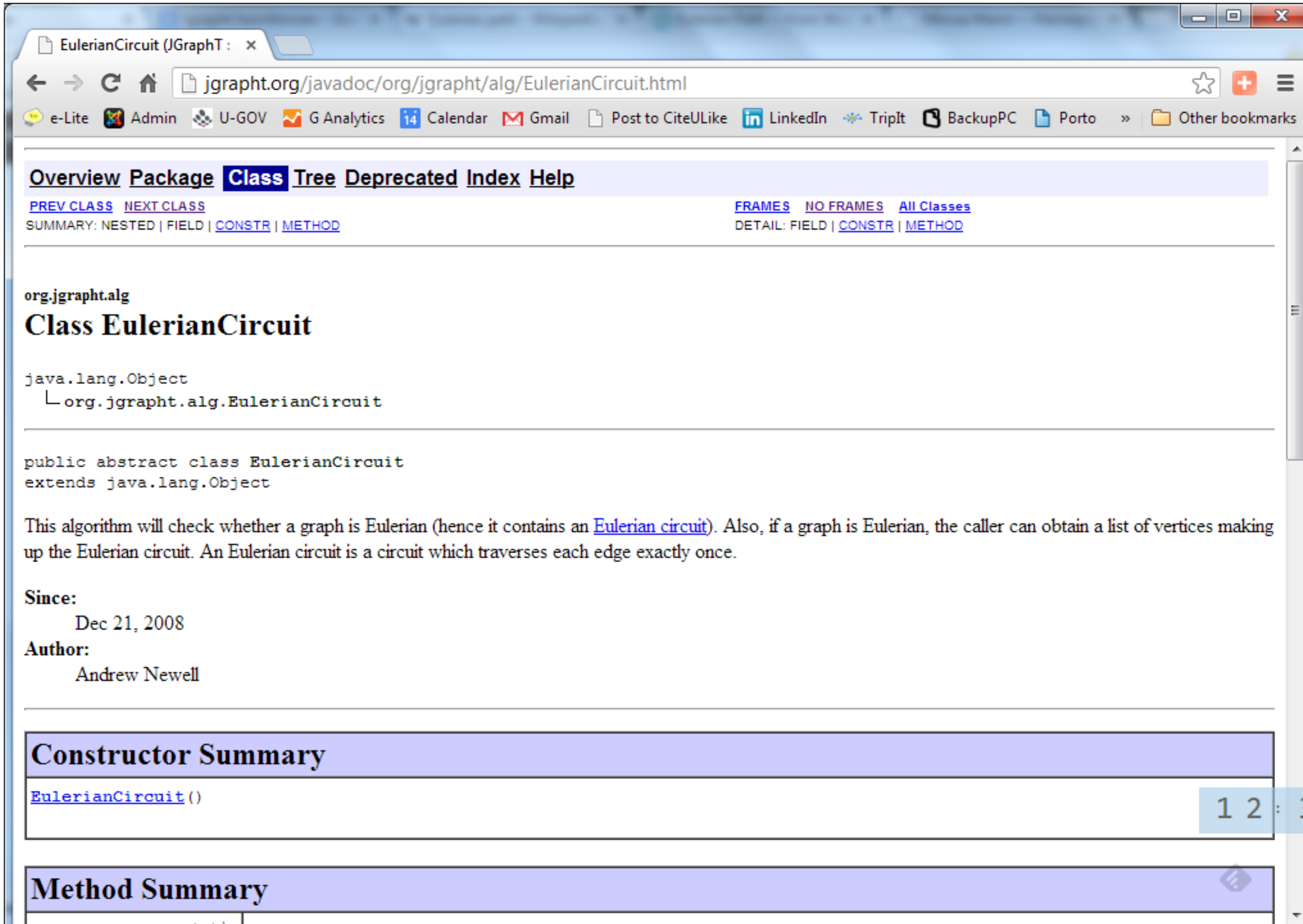


R_4: e, g, **h**, **f**, **e**, **i**, **h**, d, b, a, c, e, d, c, h, j, e

R_3: e, g, h, **d**, **b**, **a**, **c**, **e**, **d**, c, h, j, e

Q_3: h, f, e, i, h

Eulerian Circuits in JGraphT



EulerianCircuit (JGraphT :

jgrapht.org/javadoc/org/jgrapht/alg/EulerianCircuit.html

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Overview Package **Class** Tree Deprecated Index Help

PREV CLASS NEXT CLASS

FRAMES NO FRAMES All Classes

SUMMARY: NESTED | FIELD | CONSTR | METHOD

DETAIL: FIELD | CONSTR | METHOD

org.jgrapht.alg

Class EulerianCircuit

```
java.lang.Object
  └org.jgrapht.alg.EulerianCircuit
```

```
public abstract class EulerianCircuit
extends java.lang.Object
```

This algorithm will check whether a graph is Eulerian (hence it contains an Eulerian circuit). Also, if a graph is Eulerian, the caller can obtain a list of vertices making up the Eulerian circuit. An Eulerian circuit is a circuit which traverses each edge exactly once.

Since:
Dec 21, 2008

Author:
Andrew Newell

Constructor Summary
EulerianCircuit()

Method Summary

Hamiltonian Cycles

- There are theorems to identify **whether** a graph is Hamiltonian (i.e., whether it contains at least one Hamiltonian Cycle)
- **Finding** such a cycle has **no** known efficient solution, in the general case

- Example: the **Traveling Salesman Problem** (TSP)

The Traveling Salesman Problem (TSP)

Weighted or unweighted

Given a collection of cities connected by roads

Find the shortest route that visits each city exactly once.

About TSP

- Most notorious NP-complete problem.
- Typically, it is solved with a backtracking algorithm:
  - The best tour found to date is saved.
  - The search backtracks unless the partial solution is cheaper than the cost of the best tour.

What about JGraphT ?

- `org.jgrapht.alg.HamiltonianCycle`
  - ```
    static <V,E> java.util.List<V>
    getApproximateOptimalForCompleteGraph
    (SimpleWeightedGraph<V,E> g)
    ```
- But…
  - g must be a **complete** graph
  - g must satisfy the "triangle inequality": d(x,y)+d(y,z)<d(x,z)

Definition (The Metric Traveling Salesman Problem)

The metric traveling salesman problem assumes that the distance in the graph is a metric. A metric is a function $d : V \times V \to \mathbb{R}_+$ such that

- $d(x, y) + d(y, z) \geq d(x, z)$ for all $x, y, z \in V$.
- $d(x, y) = 0$ if and only if $x = y$.
    ```

ASSUMPTION: G is a metric graph.

1. Compute a minimum weight spanning tree T for G.
2. Perform a depth-first traversal of T starting from any node, and order the nodes of G as they were discovered in this traversal.

   $\Rightarrow$ a tour that is at most twice the optimal tour in G.

Resources

- http://mathworld.wolfram.com/
- http://en.wikipedia.org/wiki/Euler_cycle
- Mircea MARIN, Graph Theory and Combinatorics, Lectures 9 and 10, http://web.info.uvt.ro/~mmarin/

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