Trees

(in computer science)

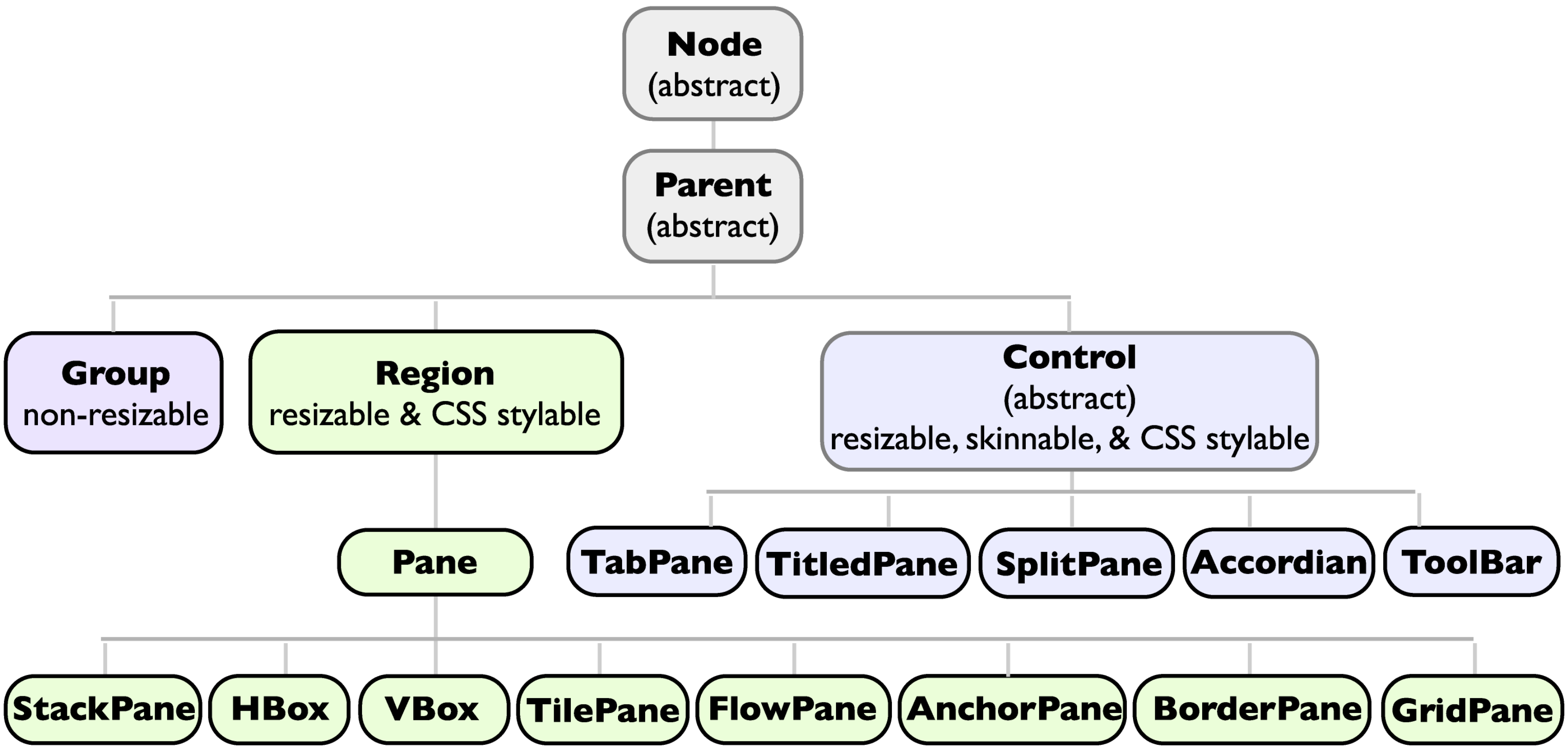
Tree in Computer Science

- A tree is a widely used data structure that simulates a hierarchical tree structure with a set of linked nodes

Tree in Computer Science

- **Fundamental** data storage structures used in programming
- Nonlinear structure
- Represents a *hierarchy*
- Items in a tree do not form a simple sequence
- Quite efficient for retrieving items (as arrays)
- Quite efficient for inserting/deleting items (as lists)

JavaFX 2.0 Layout Classes



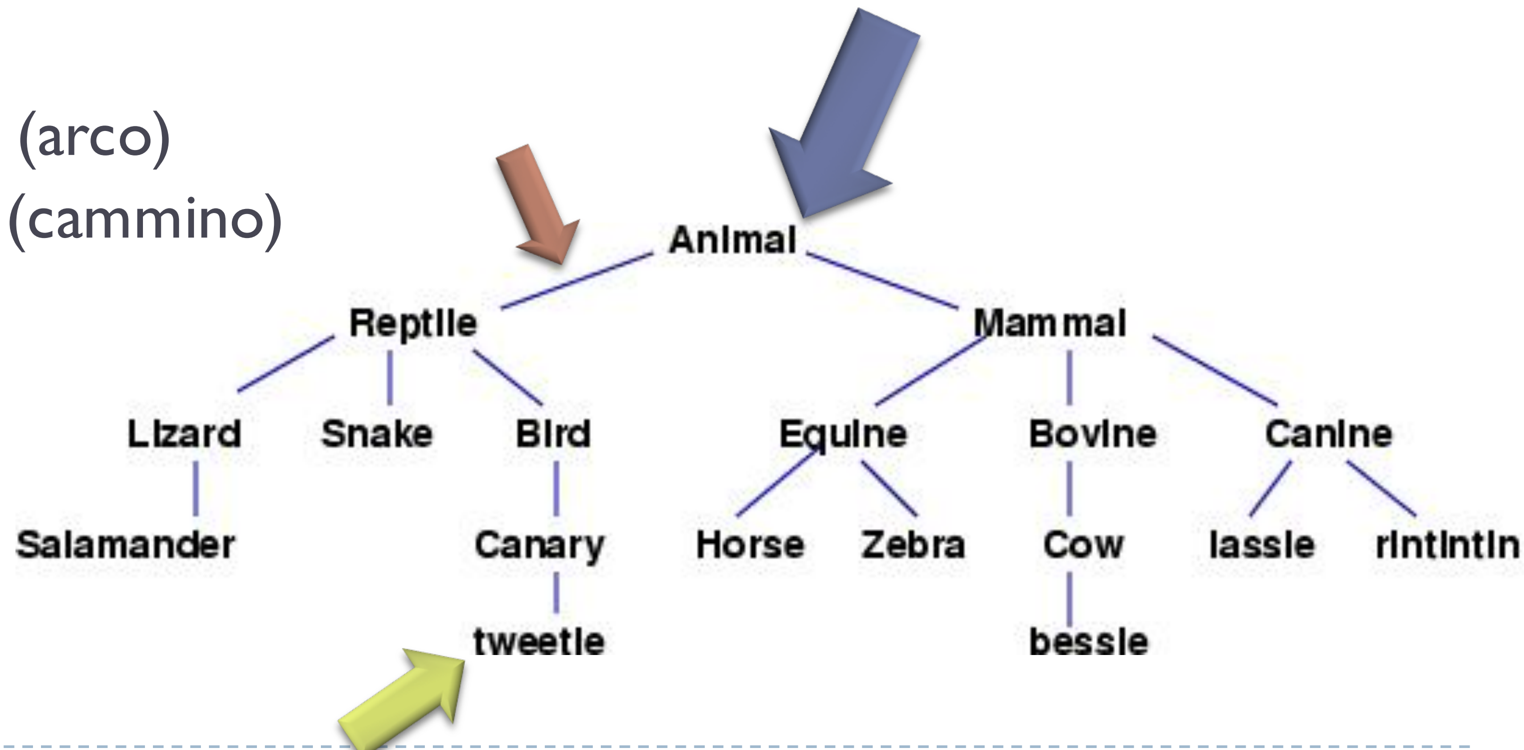
Ordinamento dello Stato Italiano

Tree basics

- Consists of nodes connected by edges
- Nodes often represent entities (complex objects)
- Edges between the nodes represent the way the nodes are related
- The only way to get from node to node is to follow a path along the edges

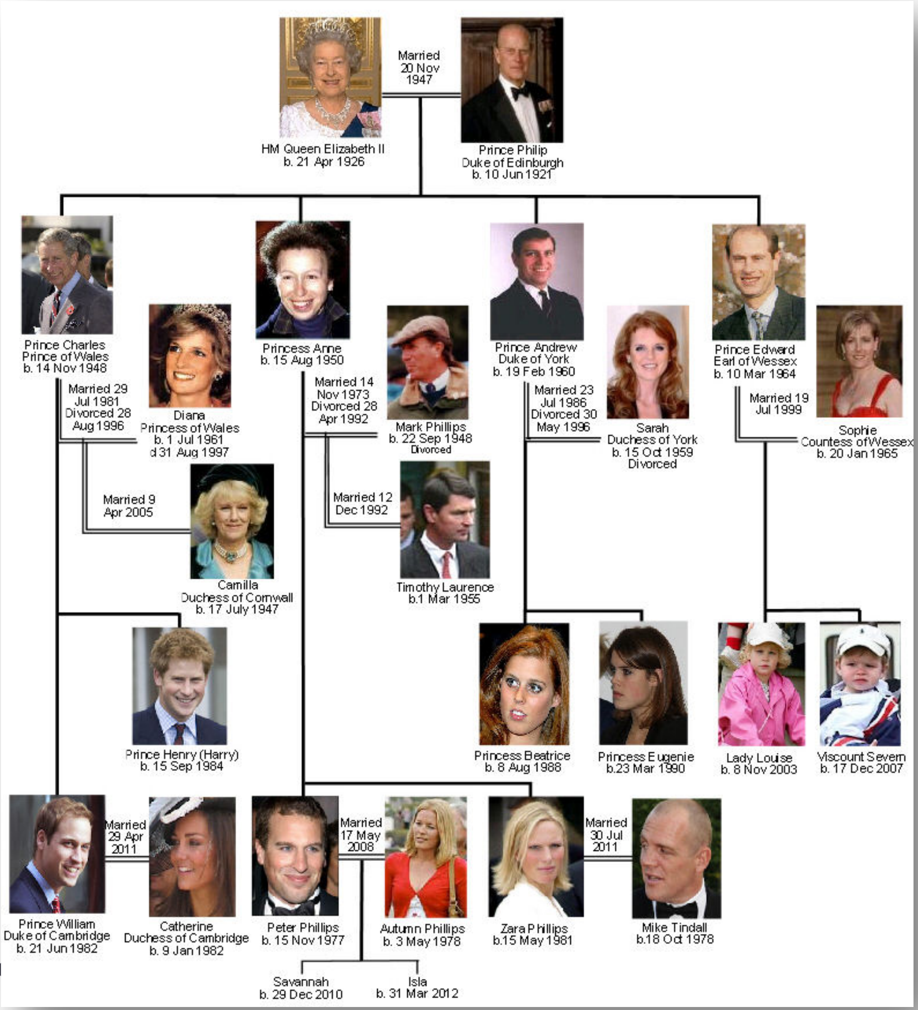
Tree Basics

- Node
  - Root (radice)
  - Leaf (foglia)
  - Interior node/branch (nodo interno)
- Links
  - Edge (arco)
  - Path (cammino)

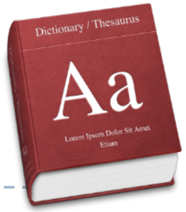


Tree Basics

- Relationship
  - Parent (padre)
  - Child nodes (nodi figli)
  - Sibling (fratelli)
  - Descendant (discendente, successore)
  - Ancestor (antenato, predecessore)

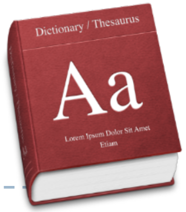


Terminology



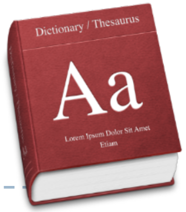
- Visiting
  - A node is visited when program control arrives at the node, usually for processing
- Traversing
  - To traverse a tree means to visit all the nodes in some specified order

Terminology



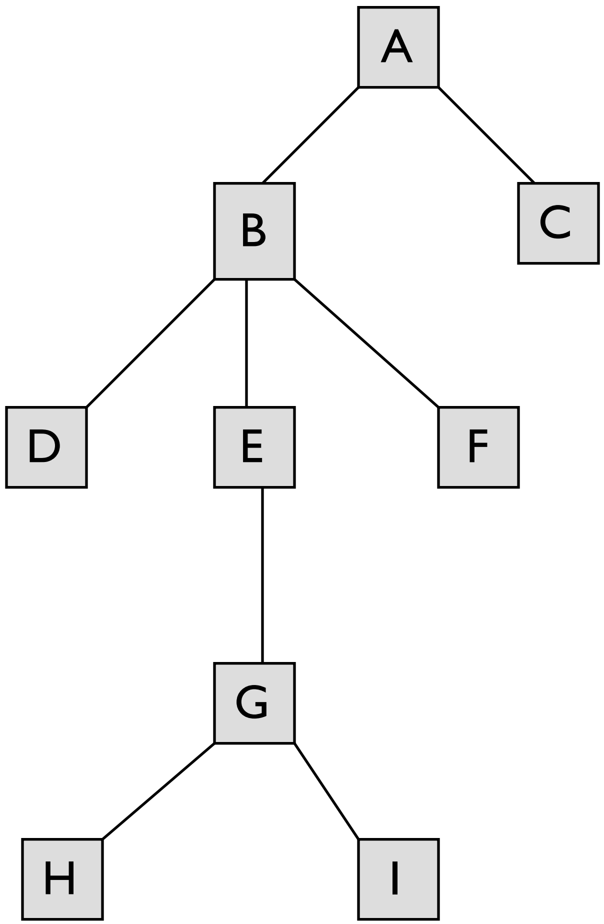
- Levels
  - The level of a particular node refers to how many generations the node is from the root
  - Root is assumed to be level 0

Terminology



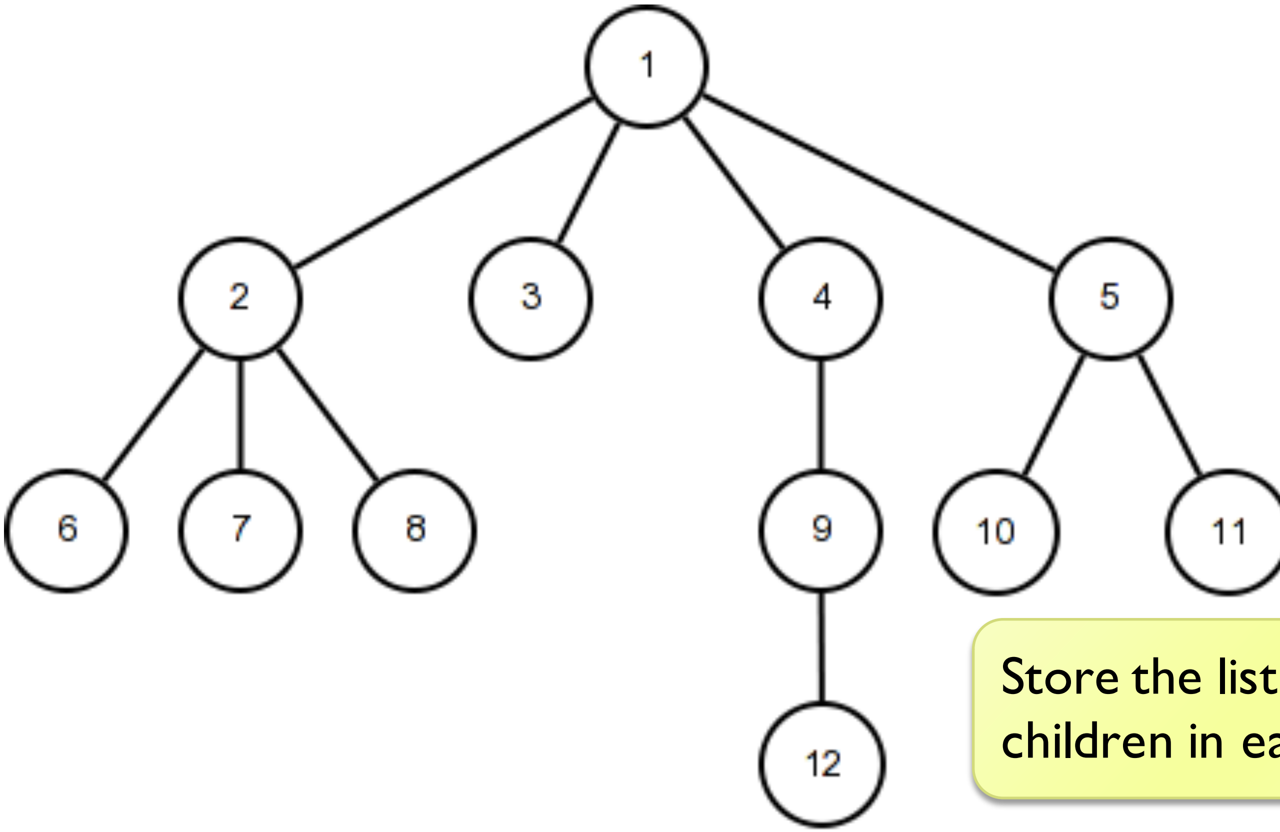
- Height
  - The height of a node is the length of the path to its farthest descendant (i.e. farthest leaf node)
  - The height of a tree is the height of the root
  - A tree with only root node has height 0

Test!



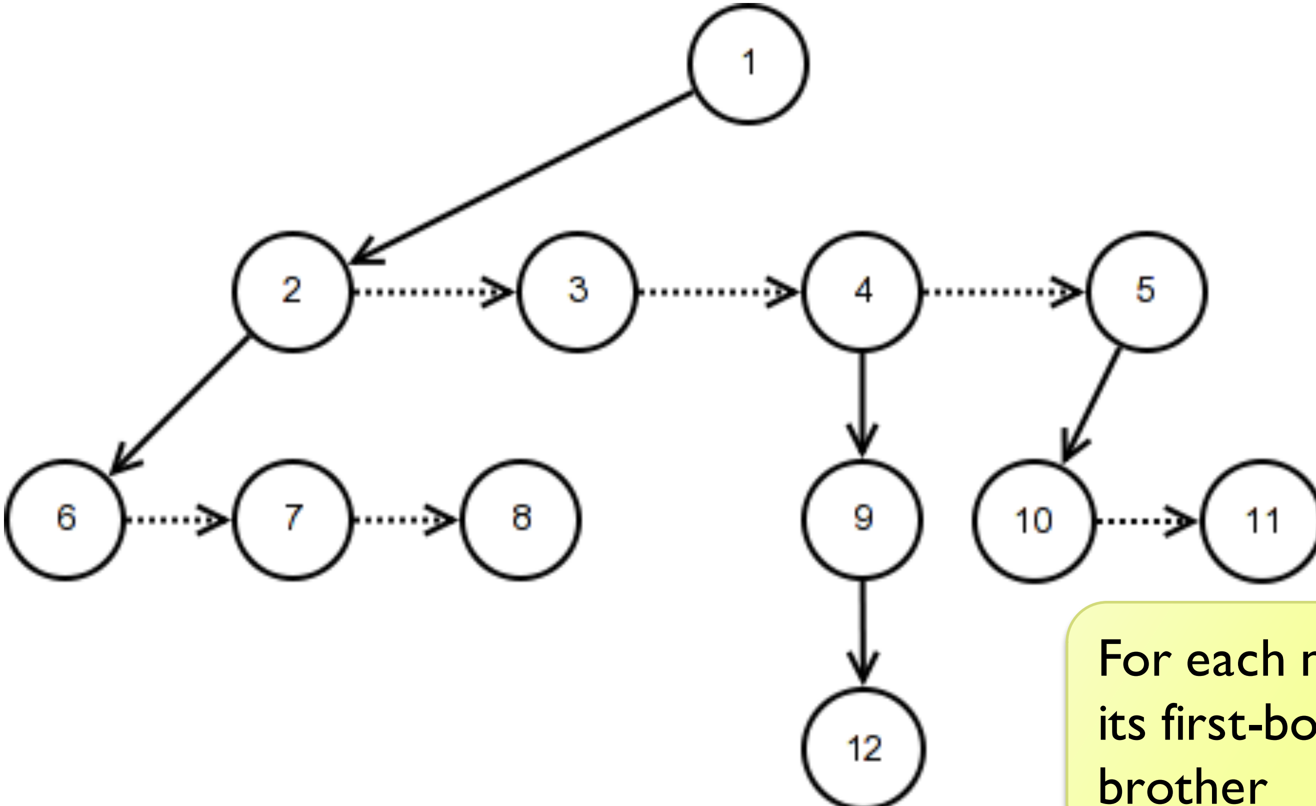
- Number of nodes
- Height
- Root Node
- Leaves
- Levels
- Interior nodes
- Ancestors of H
- Descendants of B
- Siblings of E

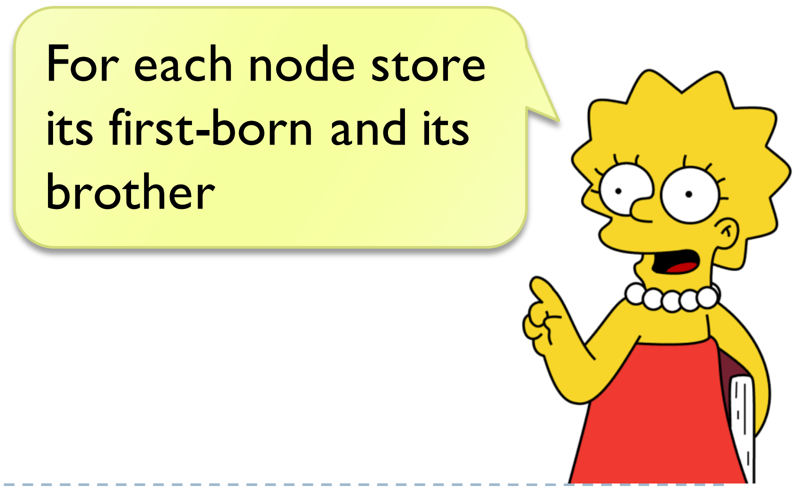
Tree representation



Store the list of children in each node

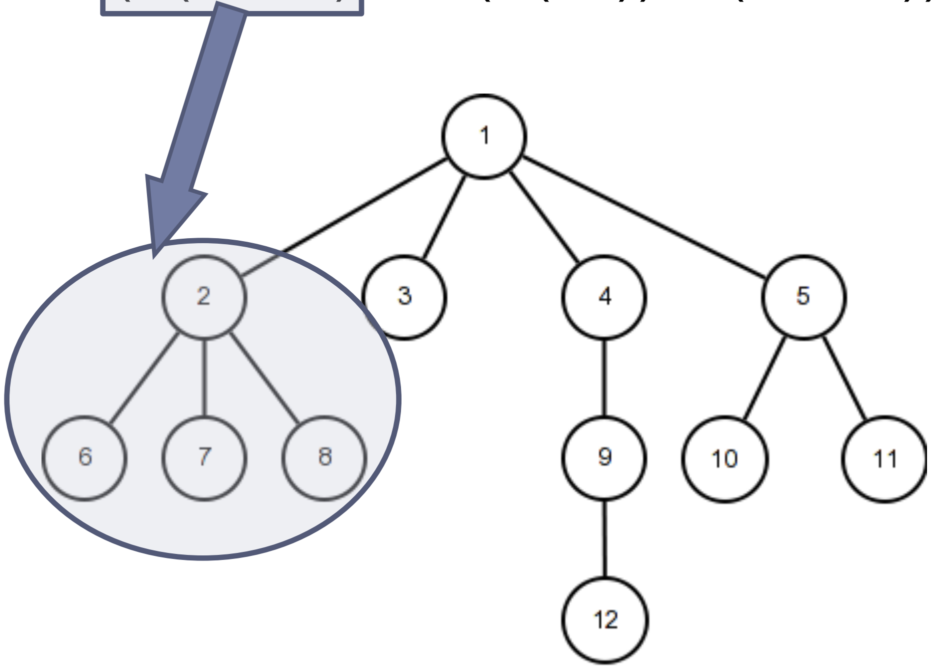
Tree representation (alt)

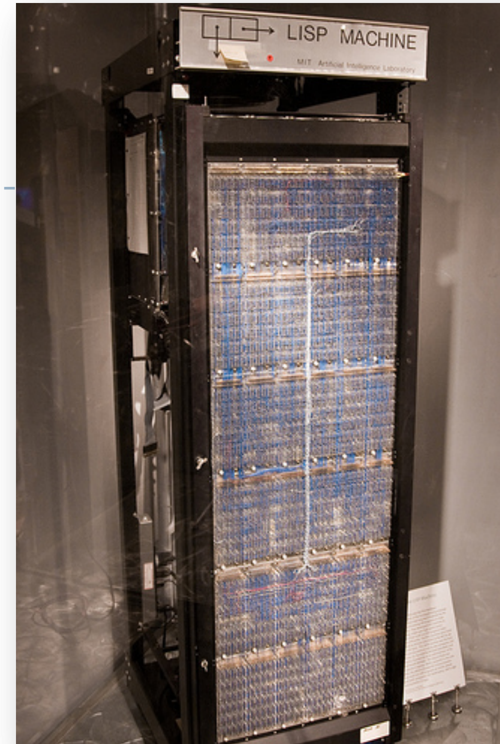




Tree representation (alt)

- 1
- **1**(2 3 4 5)
- …
- **1**(**2**(6 7 8) 3 **4**(**9**(12)) **5**(10 11))





Store each sub-tree as a separate object in a list

An oversimplified tree

```
public class Tree<T> {
   public Node<T> root;

   public Tree() {
      root = new Node<T>();
   }

   public Tree(T r) {
      root = new Node<T>(r);
   }
}
```

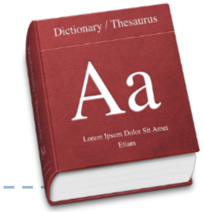
An oversimplified tree

```
public class Node<T> {
   T data;
   Node<T> parent;
   List<Node<T>> children;

   public Node() {
      data = null;
      children = new ArrayList<Node<T>>();
   }
   public Node(T d) {
      this();
      data = d;
   }

[…]
```

An oversimplified tree

```
[…]

  public void addChild(Node<T> n) {
      n.parent = this;
      children.add(n);
  }

  public void removeChild(Node<T> n) {
      children.remove(n);
  }

[…]
```

Terminology



- Visiting
  - A node is visited when program control arrives at the node, usually for processing
- Traversing
  - To traverse a tree means to visit all the nodes in some specified order

An oversimplified tree

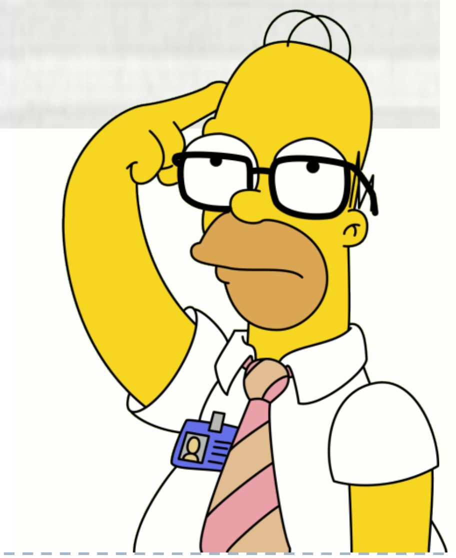
```
public class Tree<T> {
[…]

    public void Visit() {
        root.Visit();
    }
}
```

An oversimplified tree

```
public class Node<T> {
[…]
    void Visit() {
        // Do something on the node
        for(Node<T> n : children) {
            n.Visit();
        }
    }
}
```

An oversimplified tree

```
void Visit() {
   if(children.size()>0)
      System.out.print("(");
   System.out.print(data);
   for(Node<T> n : children) {
      System.out.print(" ");
      n.Visit();
   }
   if(children.size()>0)
      System.out.print(")");
}
```



An oversimplified tree

```
public class Node<T> {
[…]
    void Visit() {
        for(Node<T> n : children) {
            n.Visit();
        }
        // Do something on the node
    }
}
```

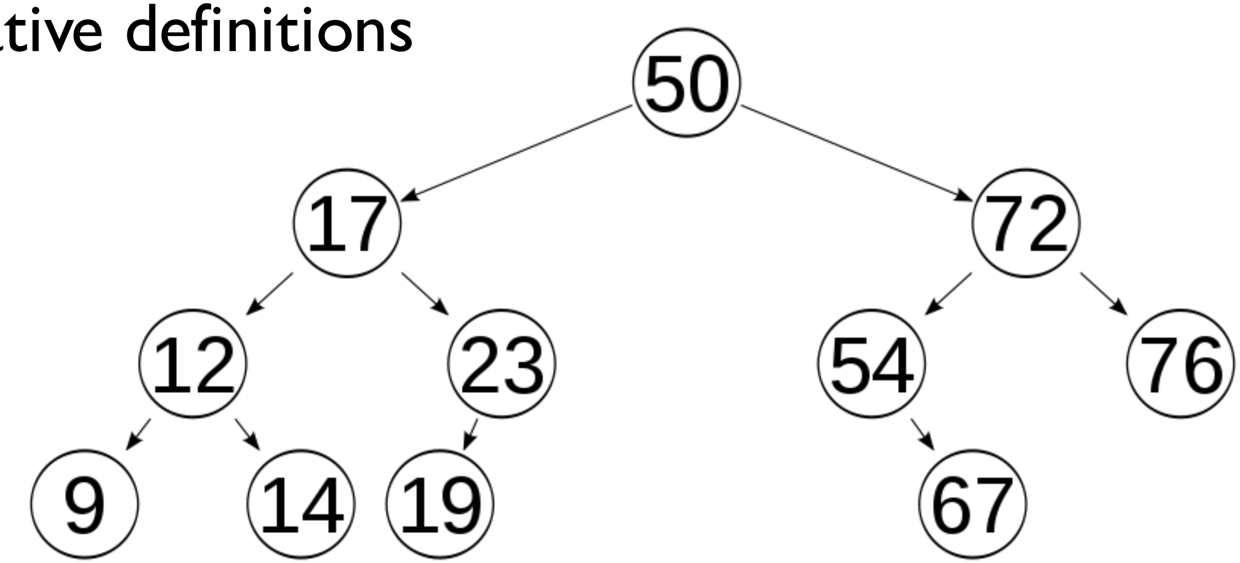
Binary Trees

Binary Tree

- A binary tree is a tree where each node has at most two children
- The two children are ordered ("left", "right")
  - Right sub-tree vs. Left sub-tree

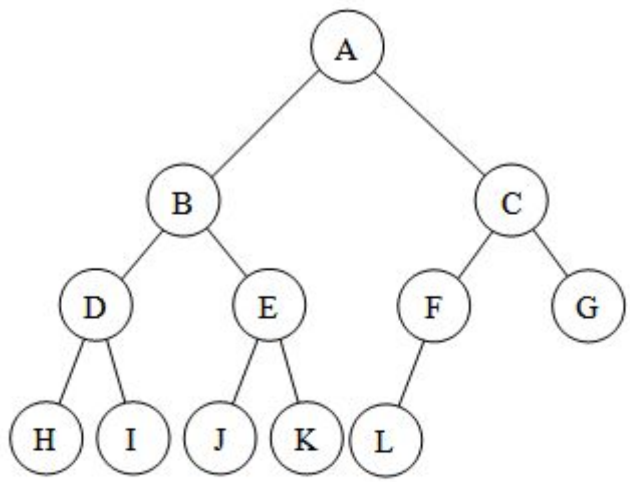
Balanced trees

- (Height-)balanced trees
  - The left and right sub-trees' heights differ by at most one
  - The two sub-trees are (height-)balanced
- Perfectly balanced
  - $2^h - 1$ nodes
- Several alternative definitions

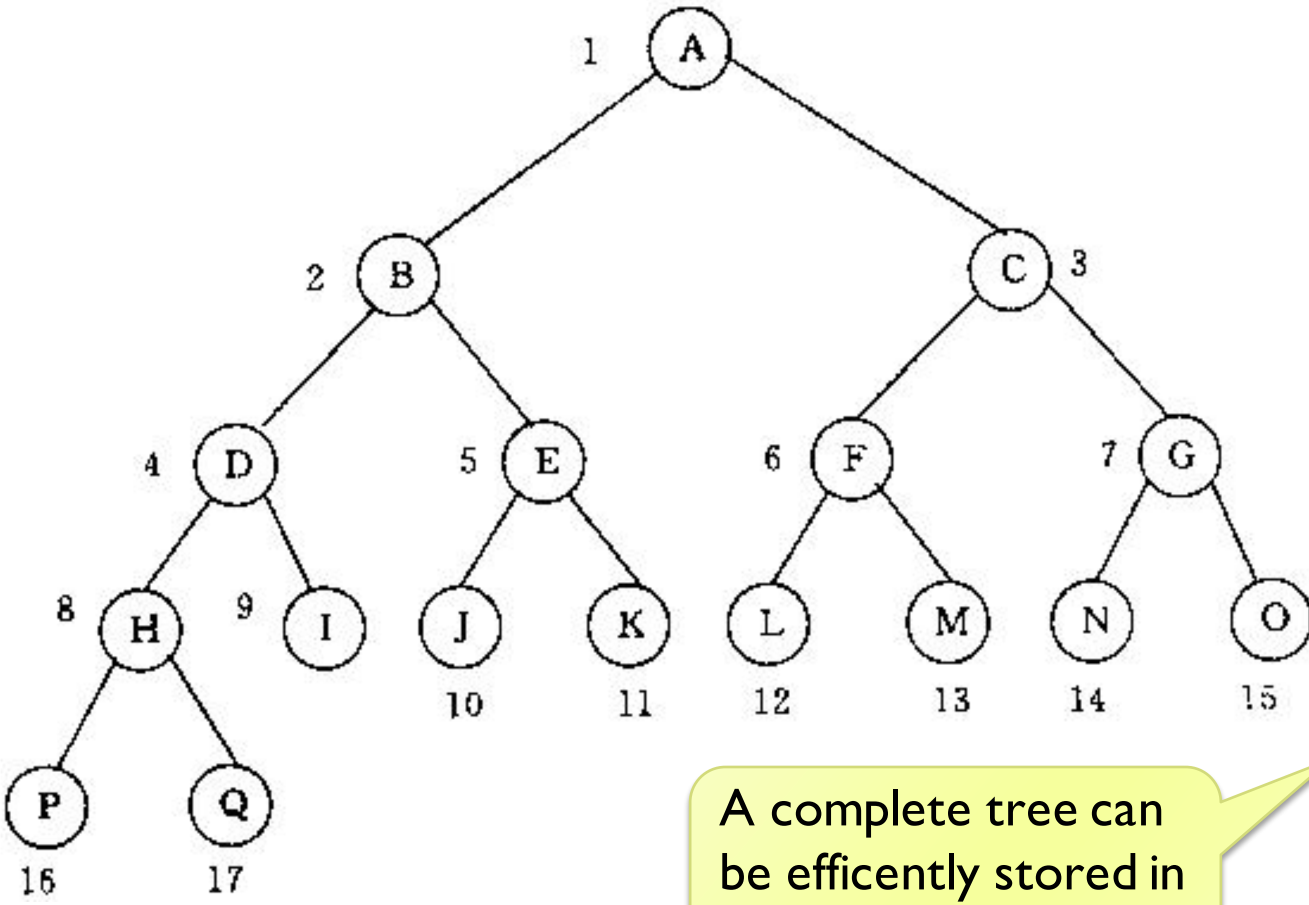


Complete trees

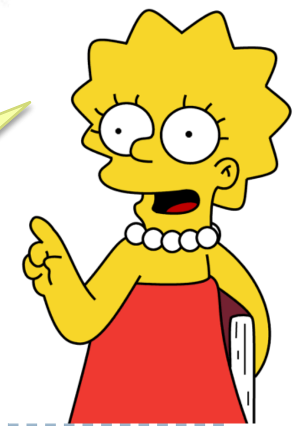
- Complete binary tree
  - Every level, except possibly the last, is completely filled, and all nodes are as far left as possible



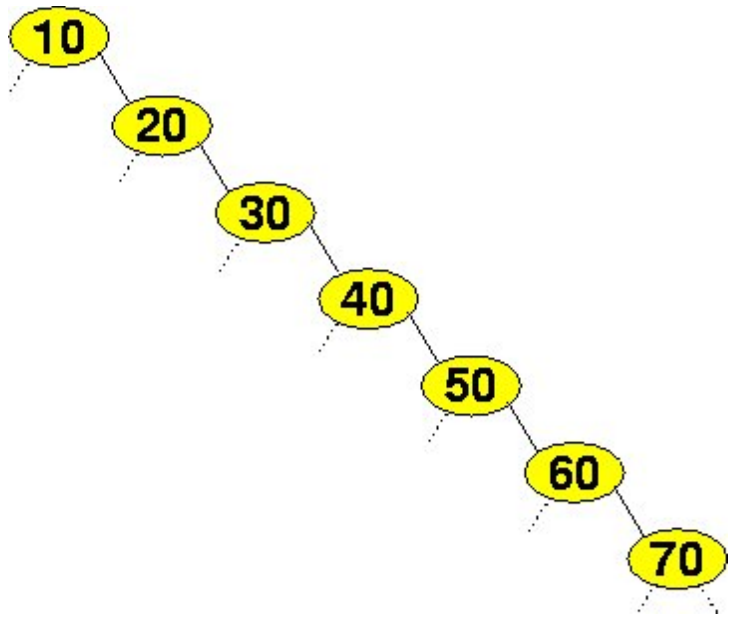
Tree representation



A complete tree can be efficently stored in an array

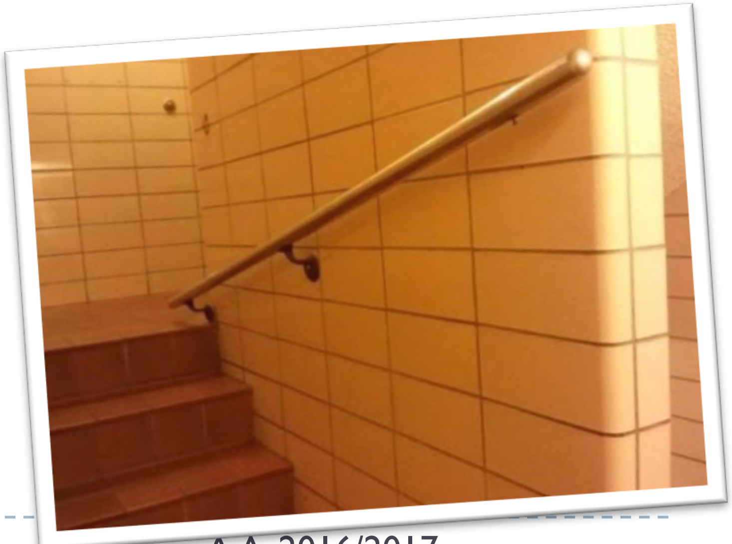


Degenerate trees

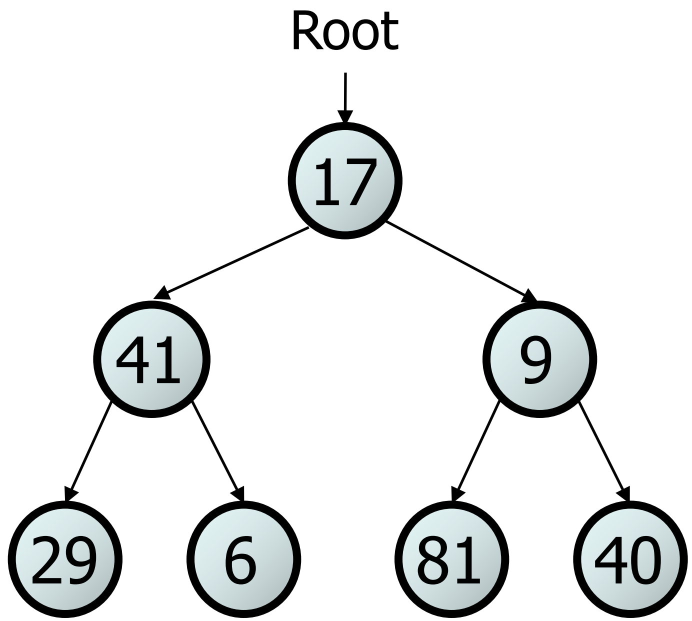


Traversal in binary trees

- Pre-order
  - process root node, then its left/right sub-trees
- In-order
  - process left sub-tree, then root node, then right
- Post-order
  - process left/right sub-trees, then root node



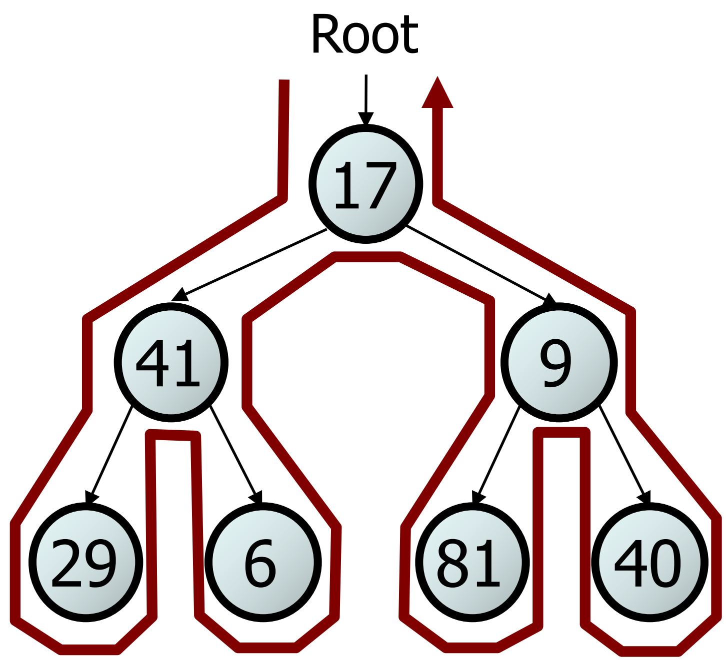
Traversal in binary trees



- **pre-order**:
- **in-order**:
- **post-order**:

Traversal trick

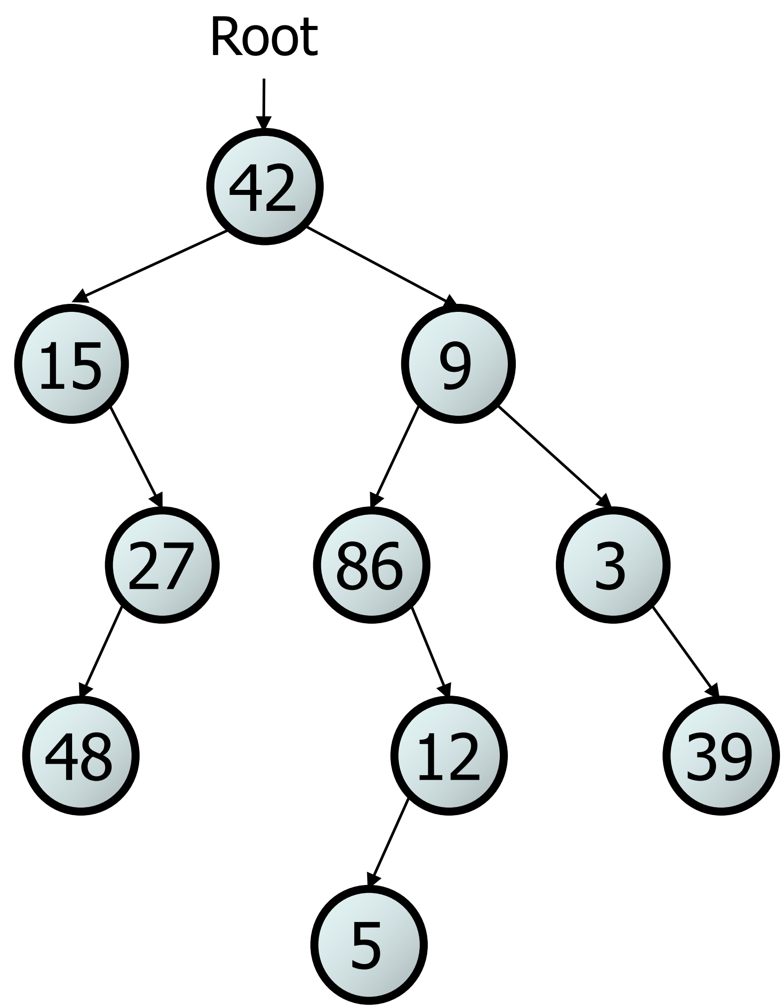
- To quickly generate a traversal:
  - Trace a path around the tree
  - As you pass a node on the proper **side**, process it
    - pre-order: left side
    - in-order: bottom
    - post-order: right side



- pre-order: 17 41 29 6 9 81 40
- in-order: 29 41 6 17 81 9 40
- post-order: 29 6 41 81 40 9 17

Exercise

- Give pre-, in-, and post-order traversals for the following tree:



pre:

in:

post:

Polish prefix notation

- Akas: "Polish notation", "prefix notation"
- Created in 1924 by the Polish logician Jan Łukasiewicz
- Operators are on the left of their operands
- If the arity of the operators is fixed
  ⇒ no need for parentheses or other brackets

- E.g.:
  - 3 * (2 + 7) ⇒ * 3 + 2 7
  - (x + y) / (2 - z) ⇒ / + x y - 2 z

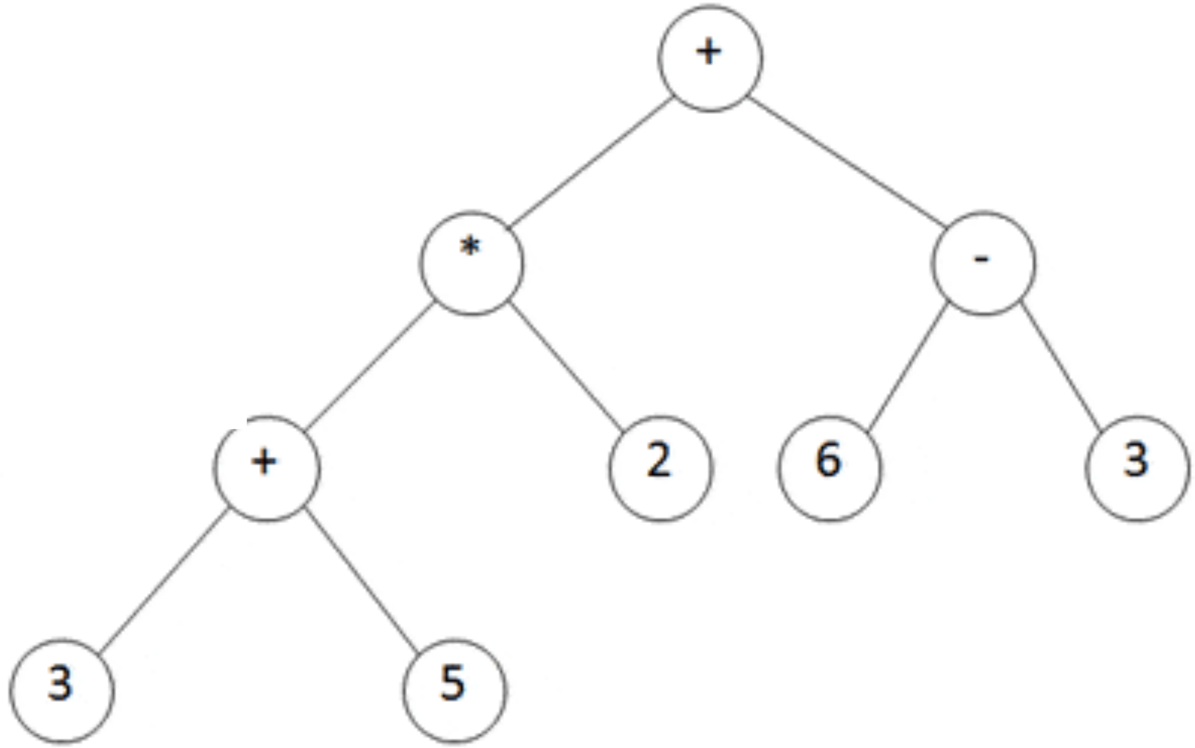
Reverse Polish notation

- (Re-)Invented by Bauer and Dijkstra in early 1960s to exploit stack for evaluating expressions
- Operator <u>follows</u> all of its operands
- If the arity of the operators is fixed
  $\Rightarrow$ no need for parentheses or other brackets

- E.g.:
  - $3 * (2 + 7) \Rightarrow 3\ 2\ 7 + *$
  - $(x + y) / (2 - z) \Rightarrow x\ y + 2\ z - /$

Traversals and notations

- In-order:
- Pre-order:
- Post-order:

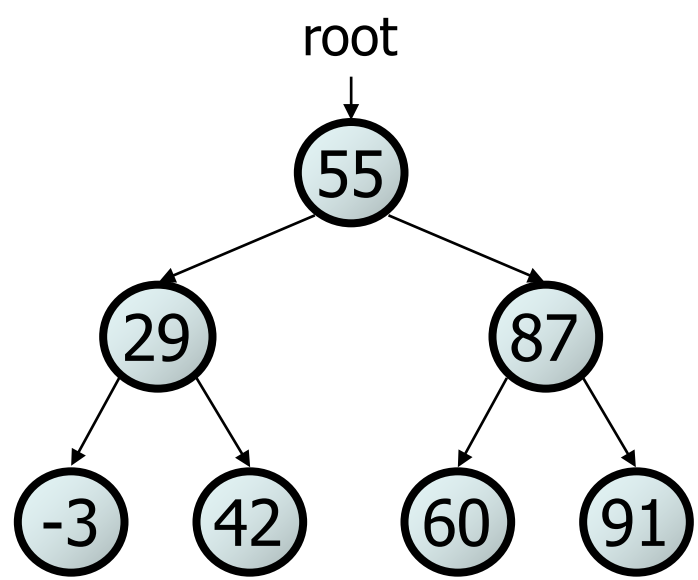


BST

Binary Search Tree

Binary search trees

- A binary tree where each non-empty node R has the following properties:
  - Elements of R's left sub-tree contain data "less than" R's data
  - Elements of R's right sub-tree contain data "greater than" R's
  - R's left and right sub-trees are also binary search trees

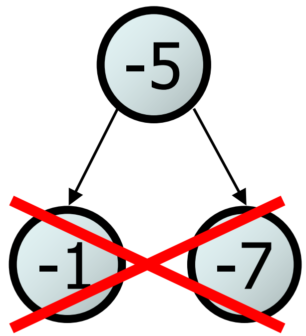


Binary search trees

- BSTs store their elements in sorted order, which is helpful for searching/sorting tasks

Exercise

- Is it a legal binary search tree?



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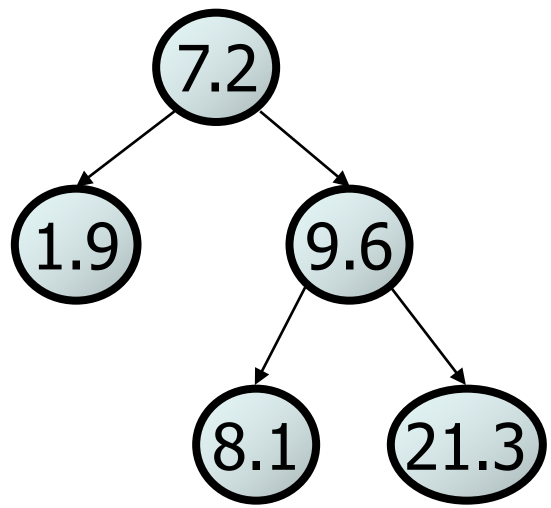
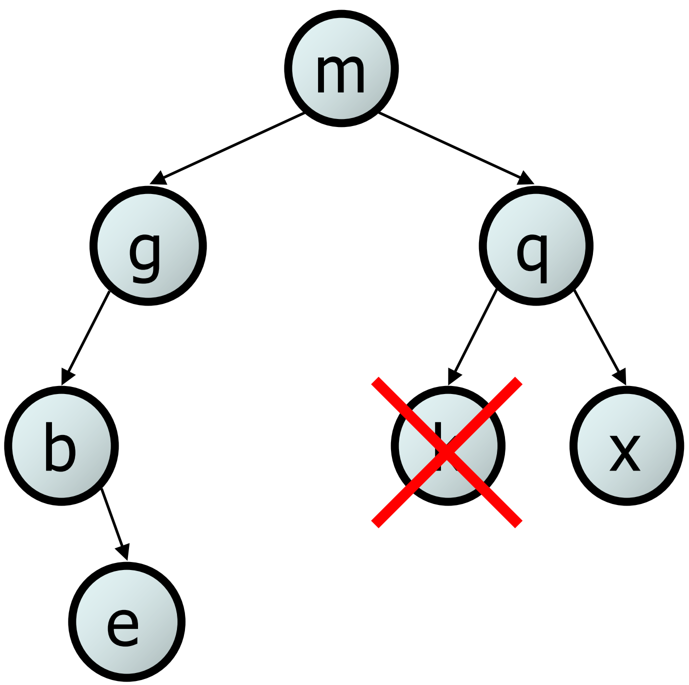


Excercise

- Is it a legal binary search tree?



Exercise

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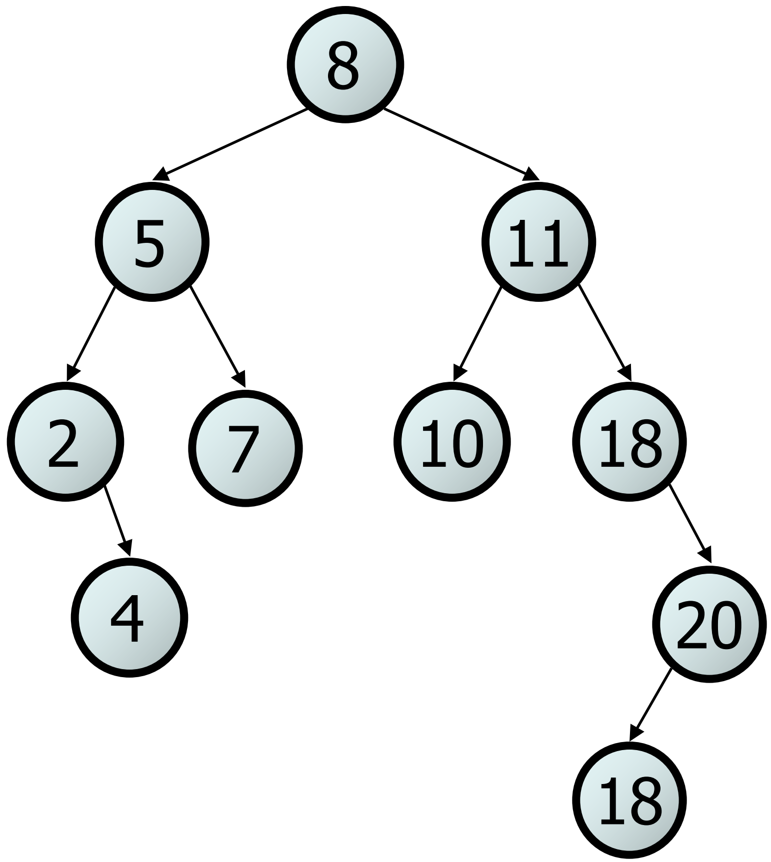
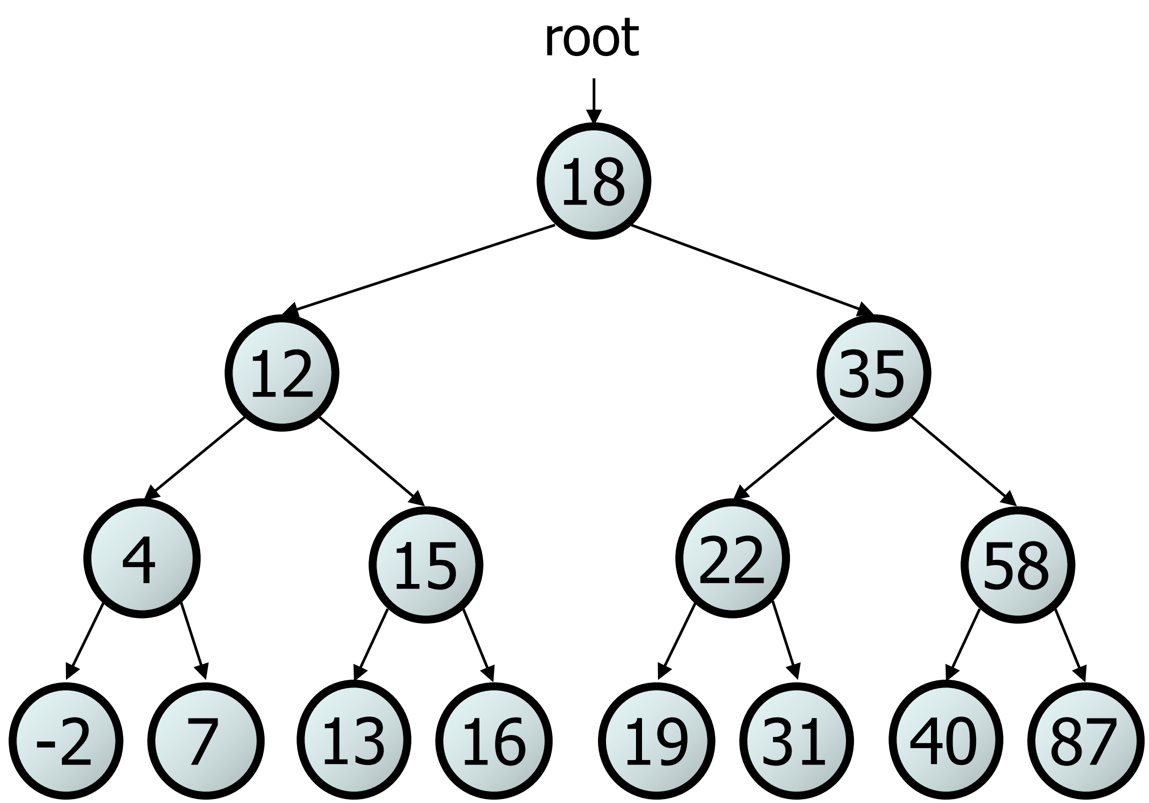


Searching in a BST

- Describe an algorithm for searching a binary search tree (try searching for 31, then 6)



Searching in a BST

- Searching in a BST is $O(h)$

  If the tree is balanced, then $h \cong \log_2 N$

  $\Rightarrow$ Searching for an element is $O(\ln N)$

Showdown

	Array	List	Hash	BST
add(element)	O(1)	O(1)	O(1)	O(ln n)
remove(object)	O(n) + O(n)	O(n) + O(1)	O(1)	O(ln n)
get(index)	O(1)	O(n)	n.a.	n.a.
set(index, element)	O(1)	O(n) + O(1)	n.a.	n.a.
add(index, element)	O(1) + O(n)	O(n) + O(1)	n.a.	n.a.
remove(index)	O(n)	O(n) + O(1)	n.a.	n.a.
contains(object)	O(n)	O(n)	O(1)	O(ln n)
indexOf(object)	O(n)	O(n)	n.a.	n.a.

Licenza d'uso