

Summary

- ▶ Graph visits
- ▶ Visits in JGraphT

Visit Algorithms

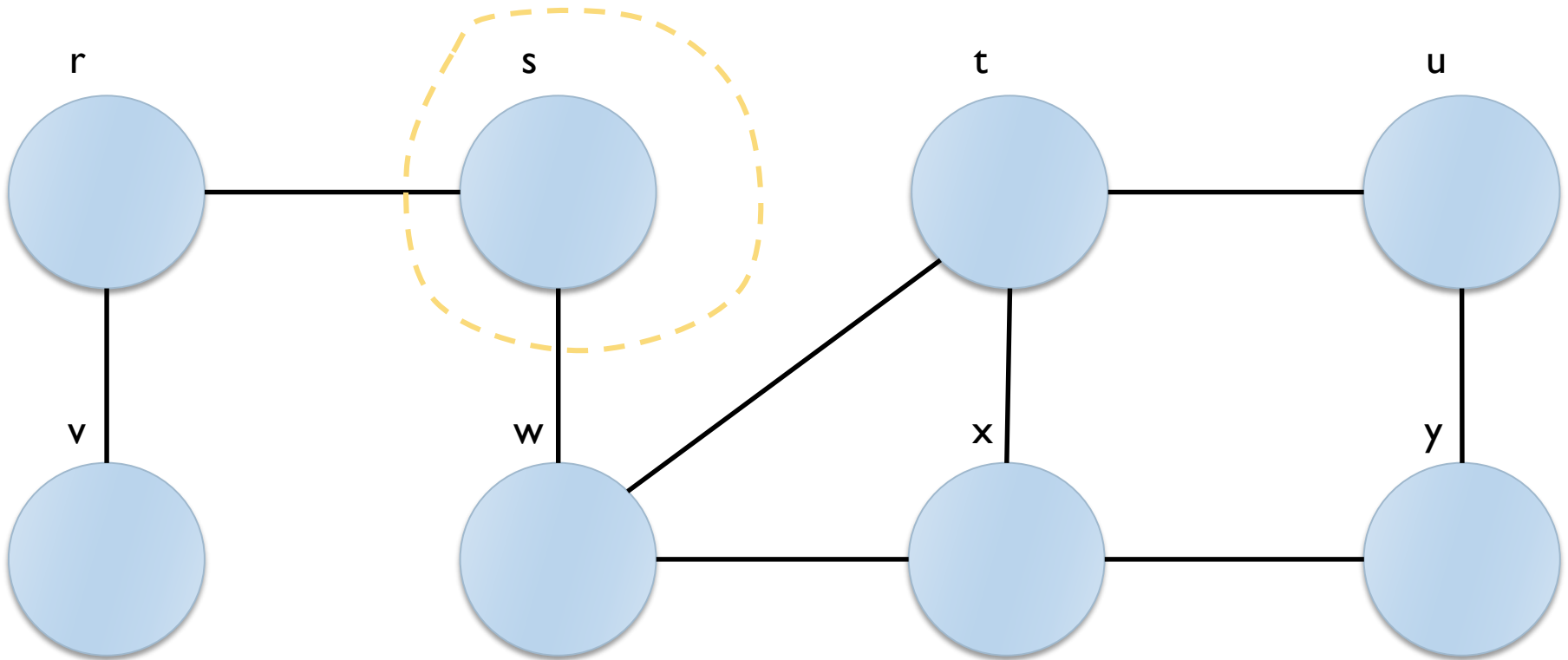
- ▶ **Visit =**
 - ▶ Systematic exploration of a graph
 - ▶ Starting from a ‘source’ vertex
 - ▶ Reaching all reachable vertices
- ▶ **Main strategies**
 - ▶ Breadth-first visit (“in ampiezza”)
 - ▶ Depth-first visit (“in profondità”)

Breadth-First Visit

- ▶ Also called Breadth-first search (BFV or BFS)
- ▶ All reachable vertices are visited “by levels”
 - ▶ L – level of the visit
 - ▶ S_L – set of vertices in level L
 - ▶ $L=0, S_0 = \{ v_{\text{source}} \}$
 - ▶ Repeat while S_L is not empty:
 - ▶ S_{L+1} = set of all vertices:
 - not visited yet, and
 - adjacent to at least one vertex in S_L
 - ▶ $L=L+1$

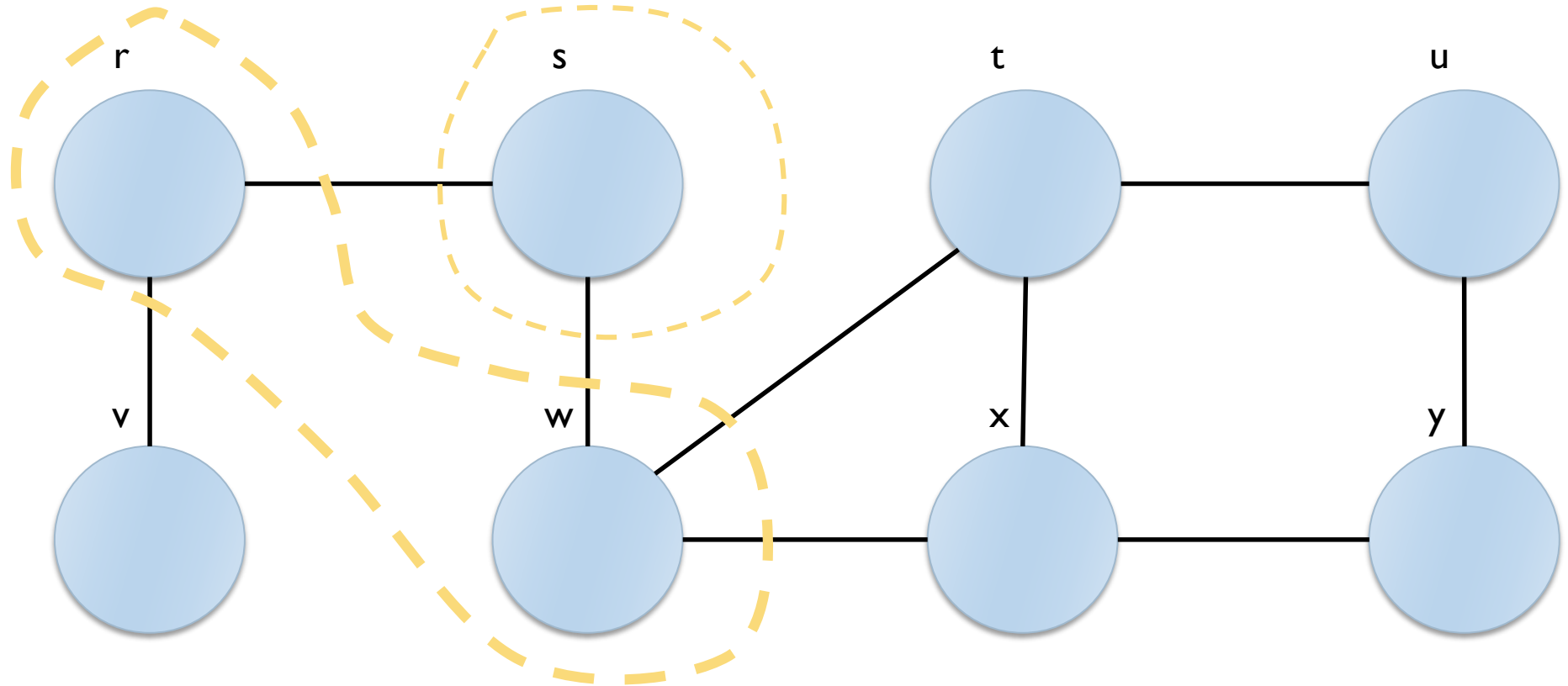
Example

Source = s
L = 0
 $S_0 = \{s\}$



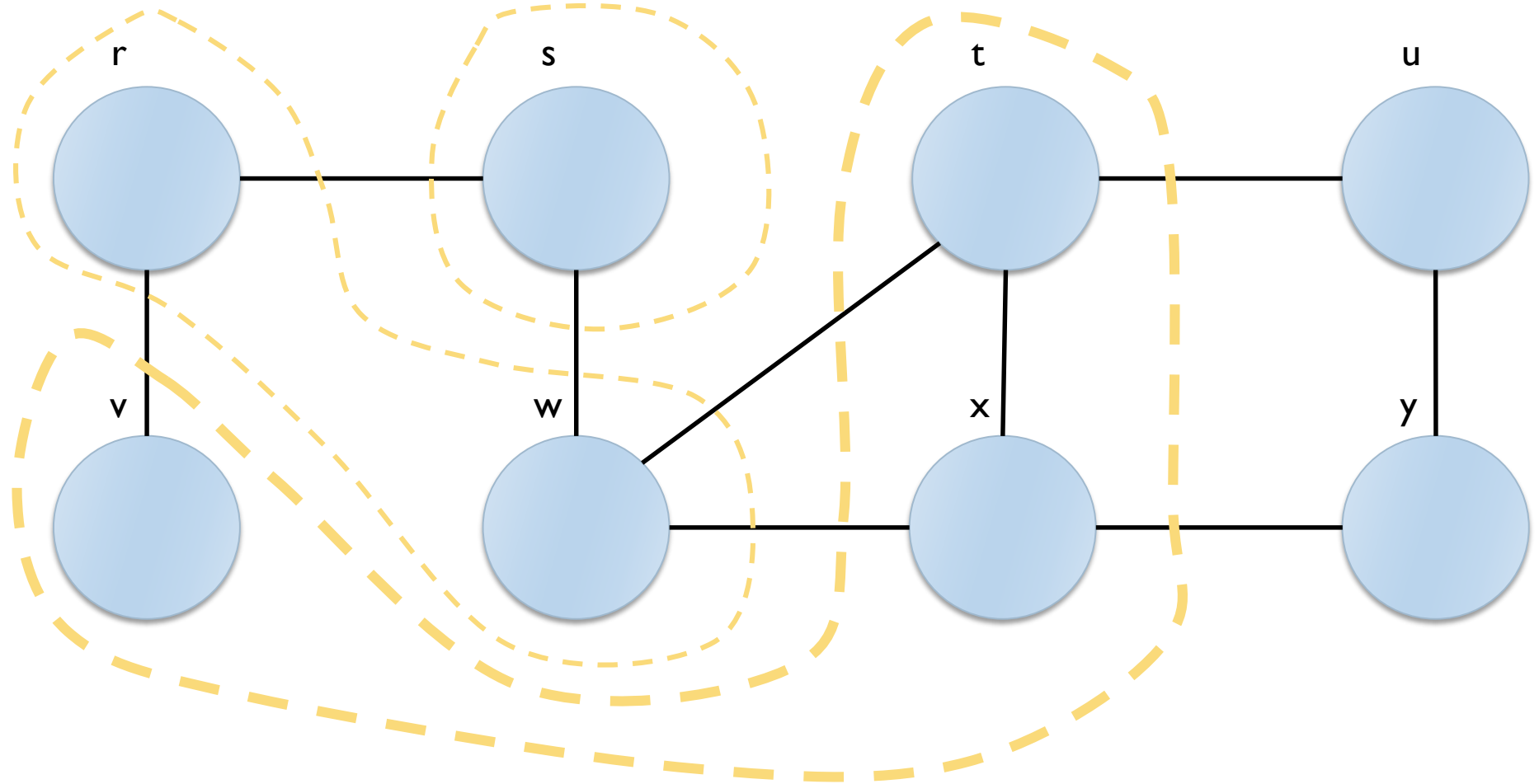
Example

$L = I$
 $S_0 = \{s\}$
 $S_1 = \{r, w\}$



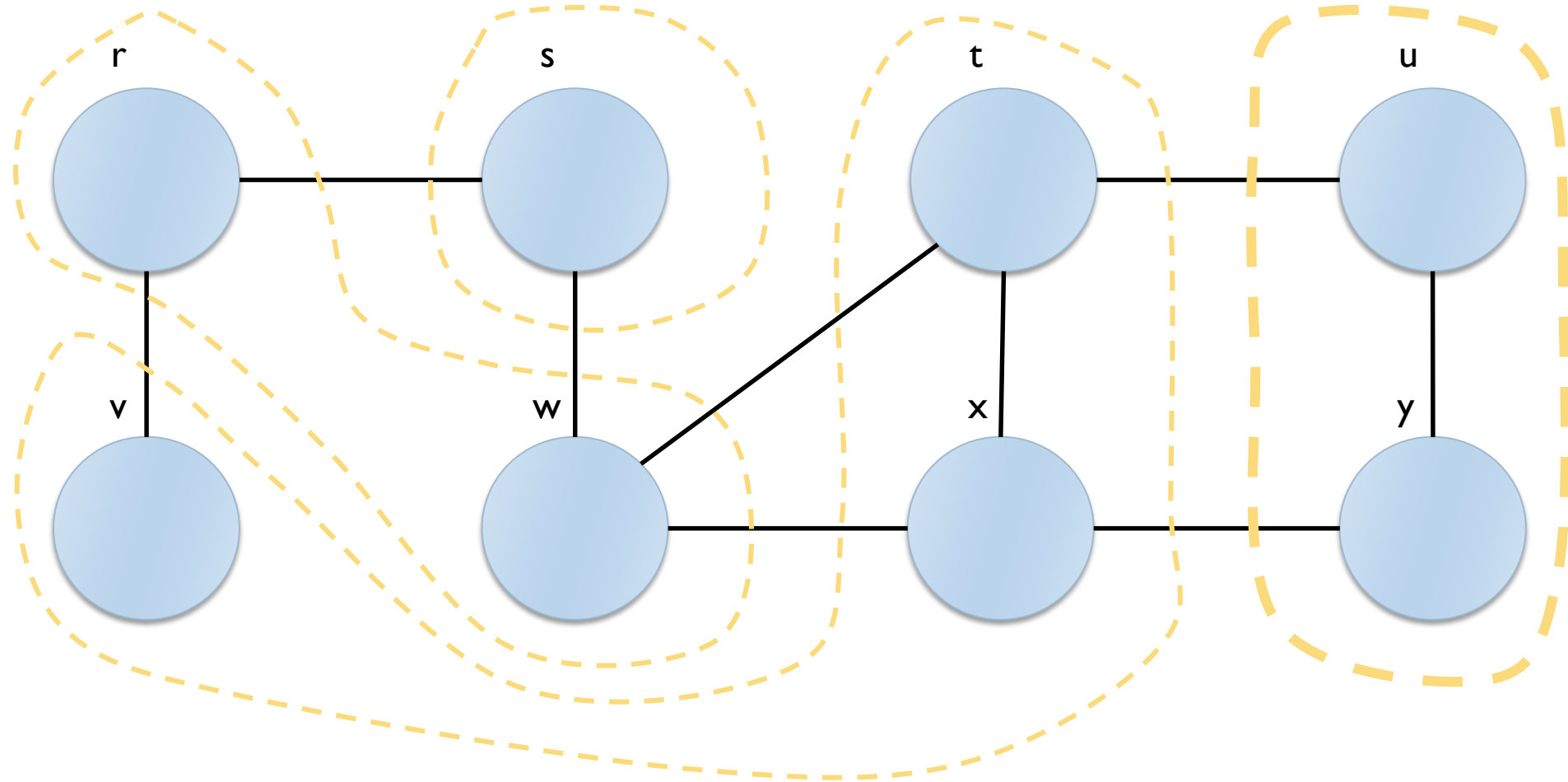
Example

$L = 2$
 $S_1 = \{r, w\}$
 $S_2 = \{v, t, x\}$



Example

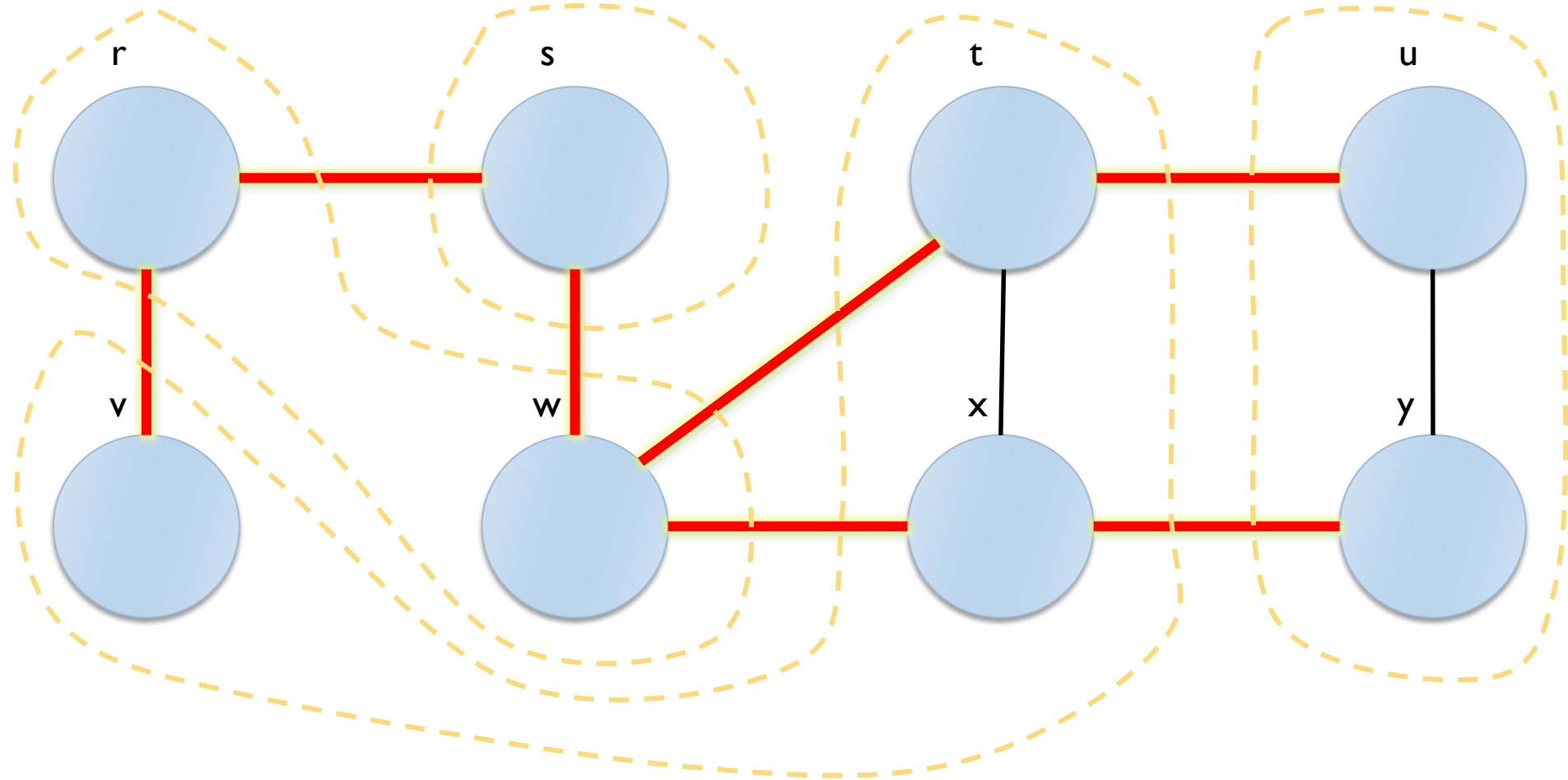
$L = 3$
 $S_2 = \{v, t, x\}$
 $S_3 = \{u, y\}$



BFS Tree

- ▶ The result of a BFV identifies a “visit tree” in the graph:
 - ▶ The tree root is the source vertex
 - ▶ Tree nodes are all graph vertices
 - ▶ (in the same connected component of the source)
 - ▶ Tree are a subset of graph edges
 - ▶ Those edges that have been used to “discover” new vertices.

BFS Tree



Minimum (shortest) paths

- ▶ Shortest path: the minimum number of edges on any path between two vertices
- ▶ The BFS procedure computes all minimum paths for all vertices, starting from the source vertex
- ▶ NB: unweighted graph : path length = number of edges

Depth First Visit

- ▶ Also called Depth-first search (DFV or DFS)
- ▶ Opposite approach to BFS
- ▶ At every step, visit one (yet unvisited) vertex, adjacent to the last visited one
- ▶ If no such vertex exist, go back one step to the previously visited vertex
- ▶ Lends itself to recursive implementation
 - ▶ Similar to tree visit procedures

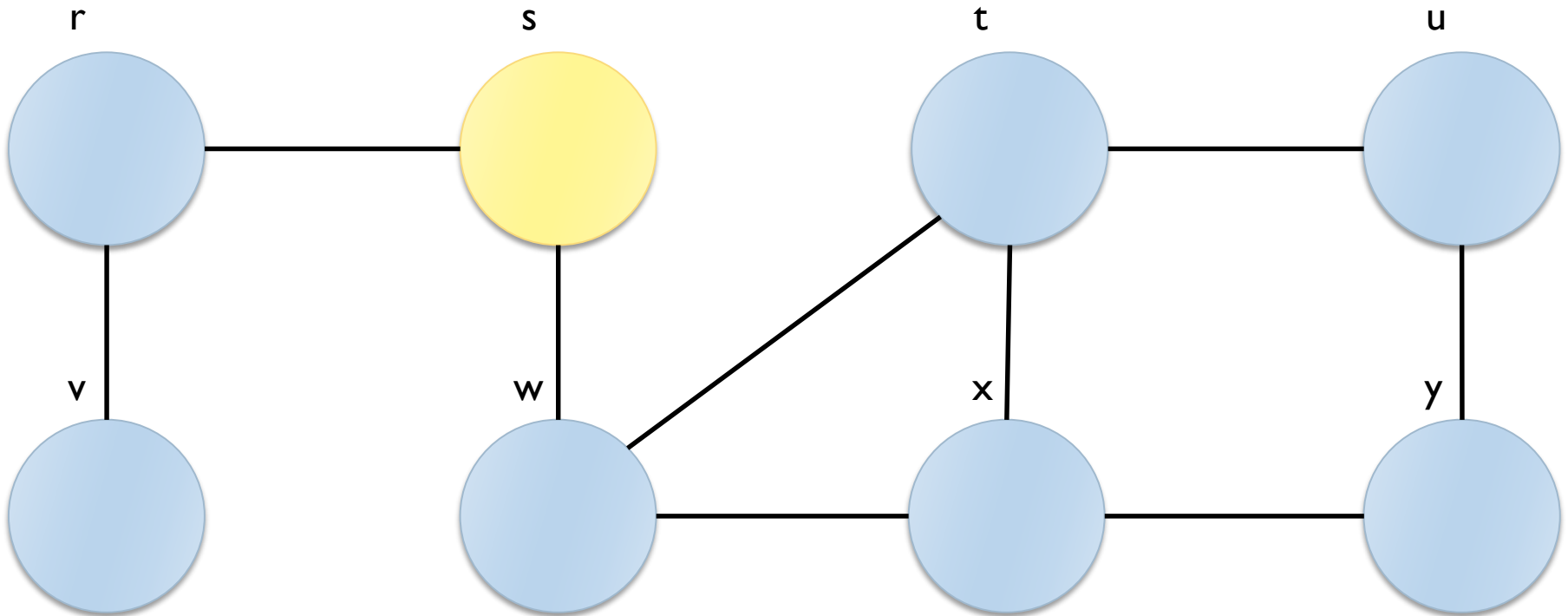
DFS Algorithm

- ▶ DFS(Vertex v)
 - ▶ For all ($w : \text{adjacent_to}(v)$)
 - ▶ If(not visited (w))
 - Visit (w)
 - DFS(w)

- ▶ Start with: DFS(source)

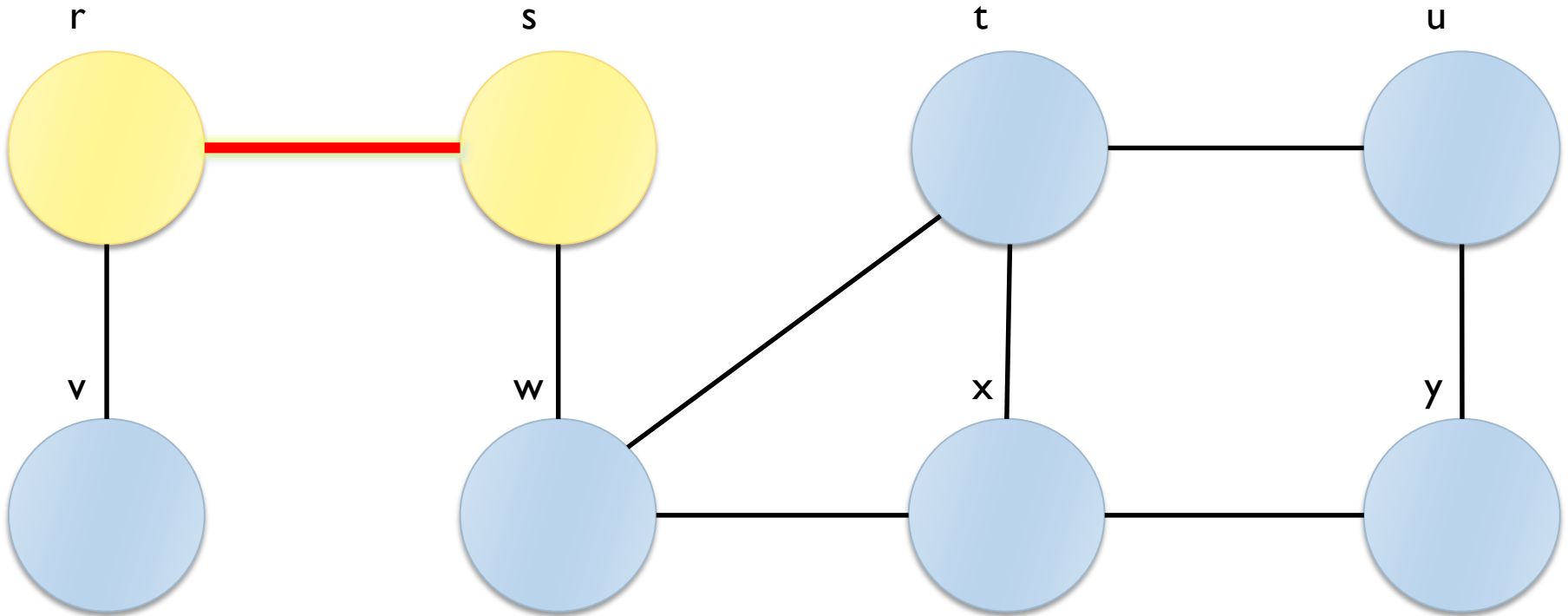
Example

Source = s



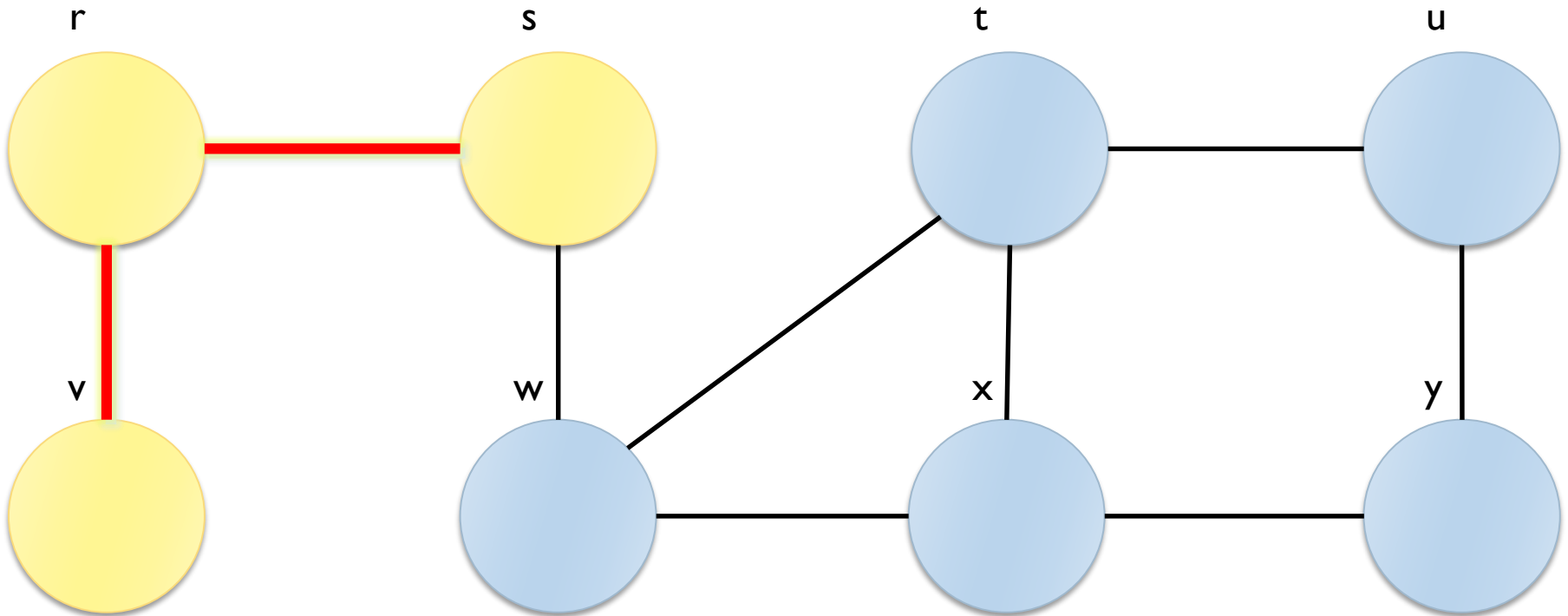
Example

Source = s
Visit r



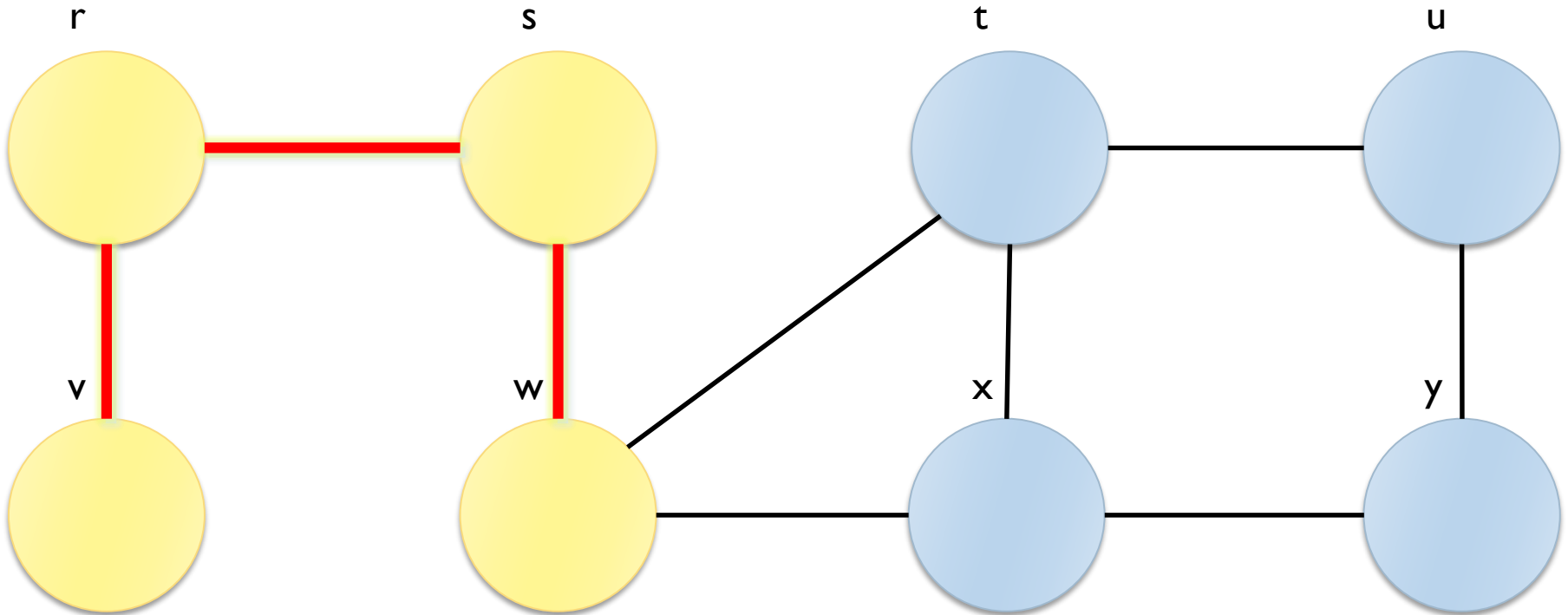
Example

Source = s
Visit r
Visit v



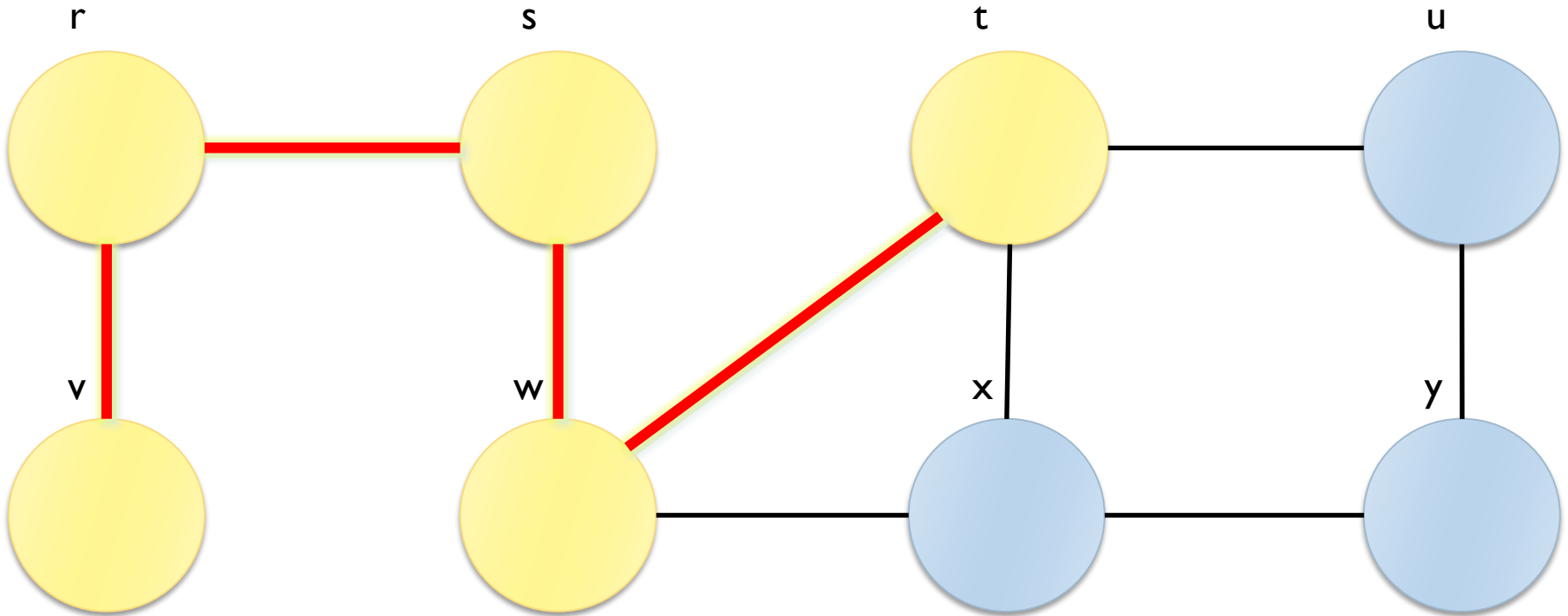
Example

Source = s
Back to r
Back to s
Visit w



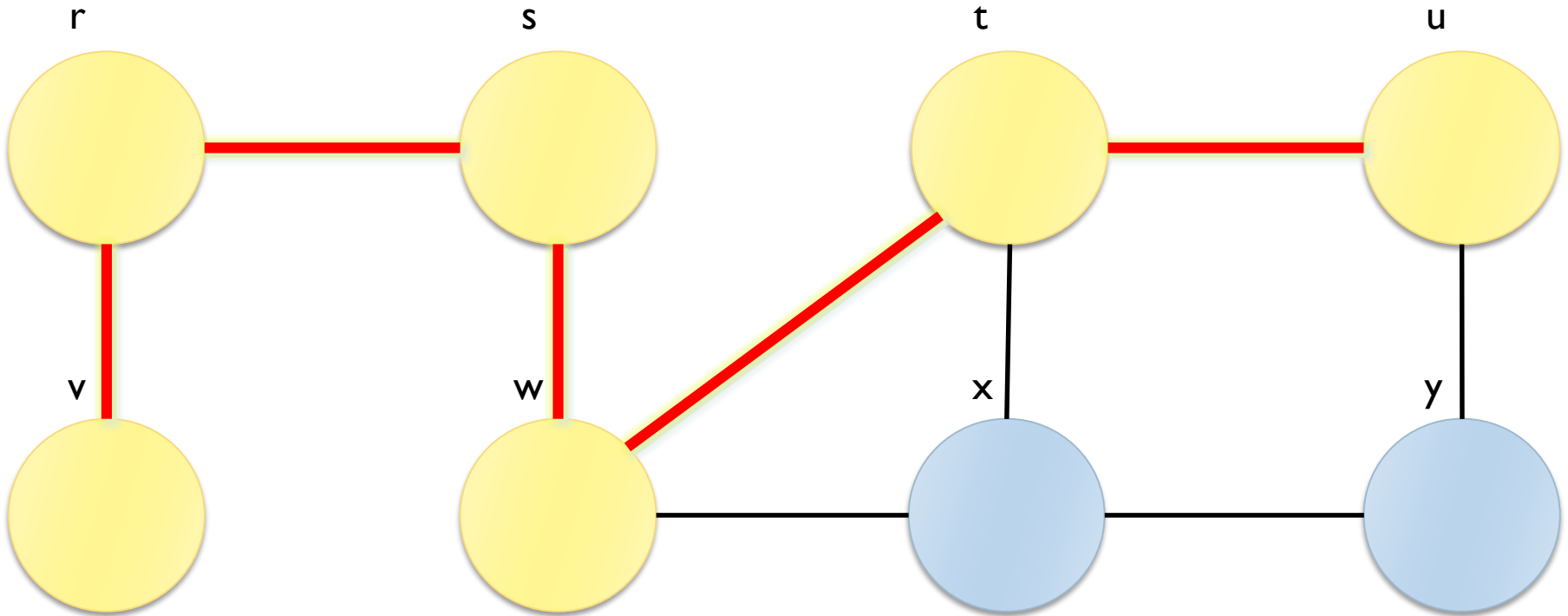
Example

Source = s
Visit w
Visit t



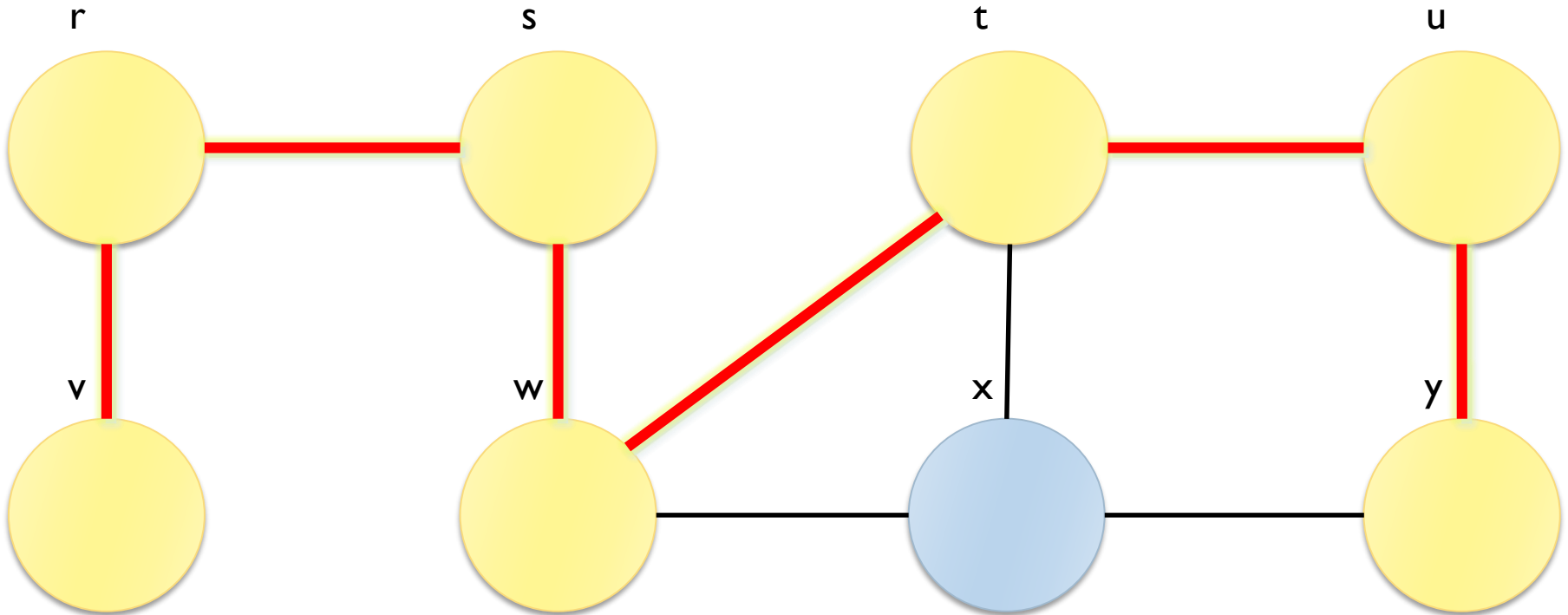
Example

Source = s
Visit w
Visit t
Visit u



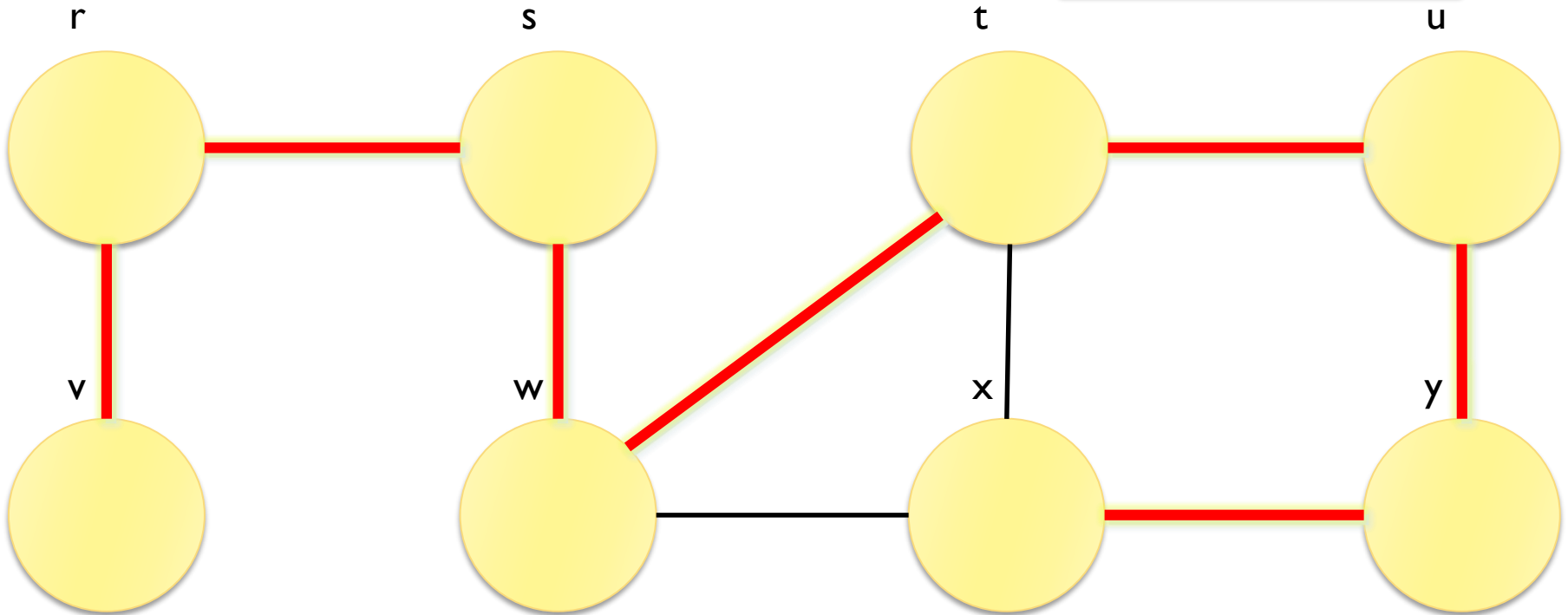
Example

Source = s
Visit w
Visit t
Visit u
Visit y

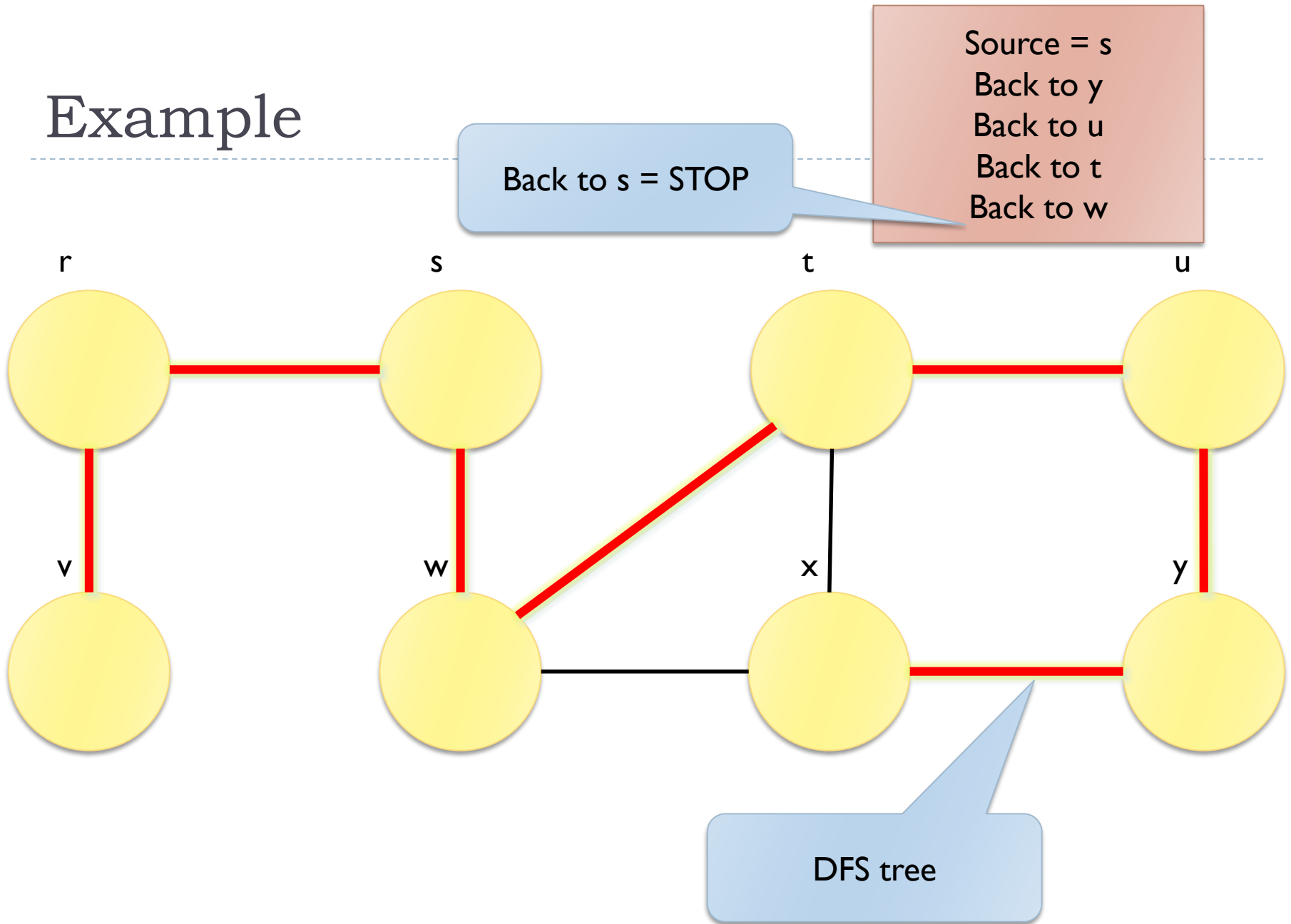


Example

Source = s
Visit w
Visit t
Visit u
Visit y
Visit x



Example

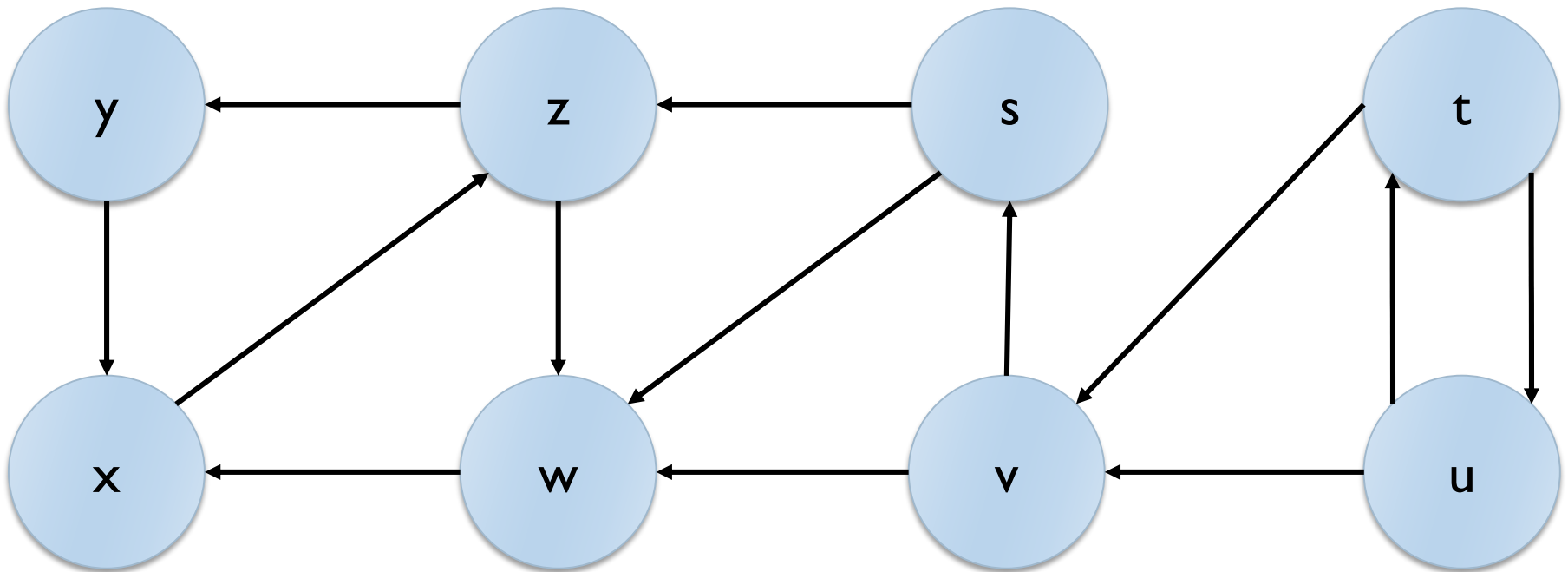


Edge classification

- ▶ In an directed graph, after a DFS visit, all edges fall in one of these 4 categories:
 - ▶ T: **Tree** edges (belonging to the DFS tree)
 - ▶ B: **Back** edges (not in T, and connect a vertex to one of its ancestors)
 - ▶ F: **Forward** edges (not in T and B, and connect a vertex to one of its descendants)
 - ▶ C: **Cross** edges (all remaining edges)

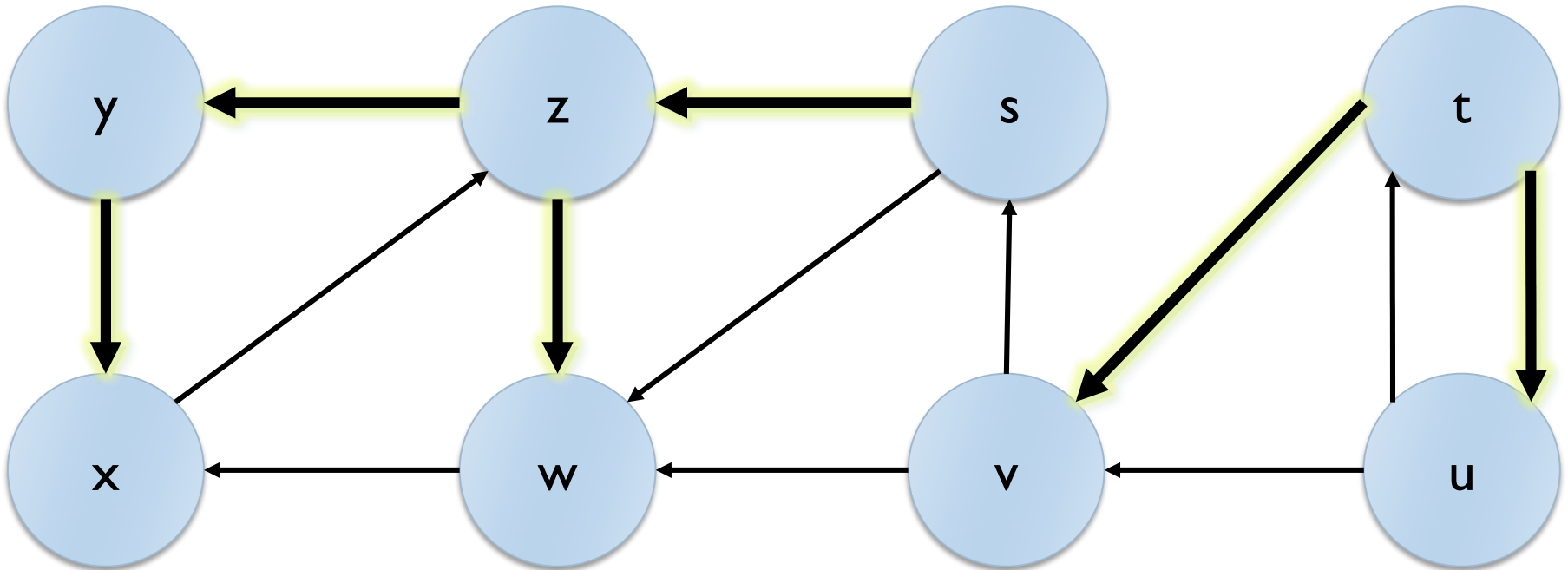
Example

Directed graph



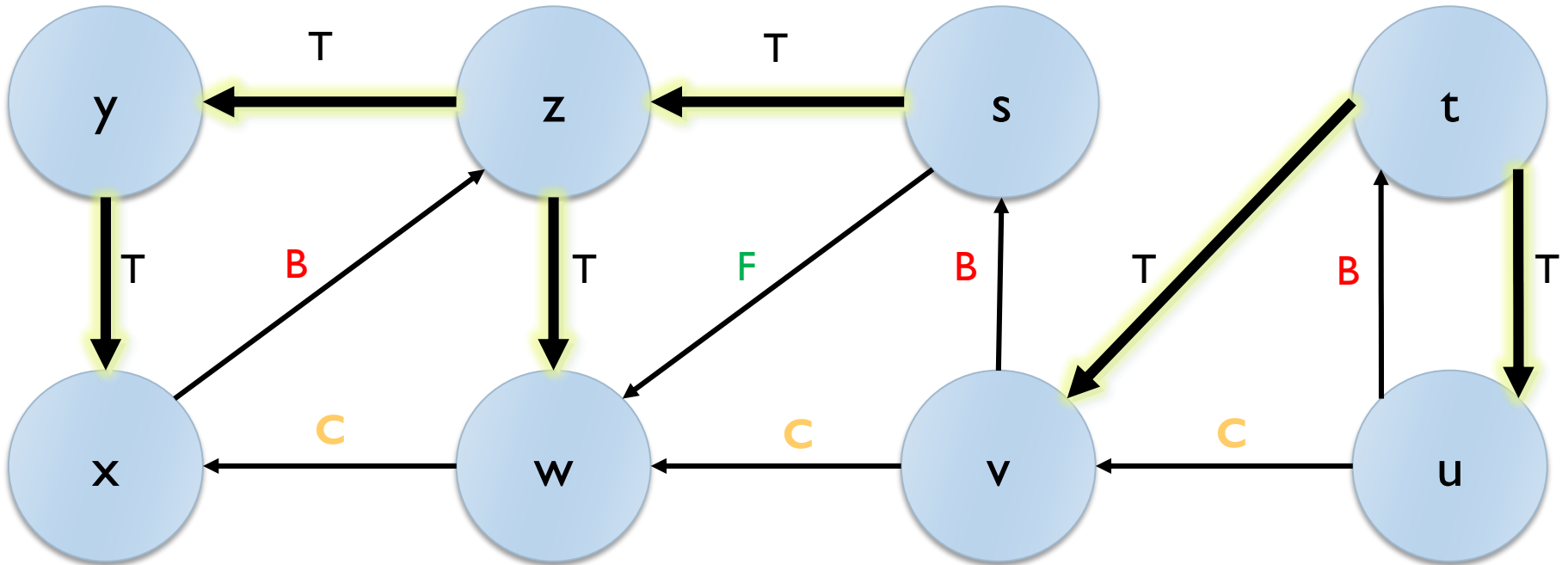
Example

DFS visit
(sources: s, t)



Example

Edge classification



Cycles

- ▶ Theorem:
- ▶ A directed graph is acyclic if and only if a depth-first visit does not produce any B edge

Complexity

- ▶ Visits have linear complexity in the graph size
 - ▶ BFS : $O(V+E)$
 - ▶ DFS : $\Theta(V+E)$
- ▶ N.B. for dense graphs, $E = O(V^2)$

Resources

- ▶ Maths Encyclopedia: <http://mathworld.wolfram.com/>
- ▶ Basic Graph Theory with Applications to Economics
<http://www.isid.ac.in/~dmishra/mpdoc/lecgraph.pdf>
- ▶ Application of Graph Theory in real world
<http://prezi.com/tsehIwvpves-/application-of-graph-theory-in-real-world/>

Resources

- ▶ Open Data Structures (in Java), Pat Morin,
<http://opendatastructures.org/>
- ▶ Algorithms Course Materials, Jeff Erickson,
<http://www.cs.uiuc.edu/~jeffe/teaching/algorithms/>
- ▶ Graphbook - A book on algorithmic graph theory, David Joyner, Minh Van Nguyen, and David Phillips,
<https://code.google.com/p/graphbook/>

JGraphT and visits

- ▶ Visits are called “traversals”
- ▶ Implemented through **Iterator** classes
- ▶ Package **org.jgrapht.traverse**

Graph traversal classes

Package org.jgrapht.traverse

Graph traversal means.

See: [Description](#)

Interface Summary

Interface	Description
<code>GraphIterator<V,E></code>	A graph iterator.

Class Summary

Class	Description
<code>AbstractGraphIterator<V,E></code>	An empty implementation of a graph iterator to minimize the effort required to implement graph iterators.
<code>BreadthFirstIterator<V,E></code>	A breadth-first iterator for a directed or undirected graph.
<code>ClosestFirstIterator<V,E></code>	A closest-first iterator for a directed or undirected graph.
<code>CrossComponentIterator<V,E,D></code>	Provides a cross-connected-component traversal functionality for iterator subclasses.
<code>DegeneracyOrderingIterator<V,E></code>	A degeneracy ordering iterator.
<code>DepthFirstIterator<V,E></code>	A depth-first iterator for a directed or undirected graph.
<code>RandomWalkIterator<V,E></code>	A random walk iterator for a directed or undirected graph.
<code>TopologicalOrderIterator<V,E></code>	A topological ordering iterator for a directed acyclic graph.

Graph iterators

- ▶ Usual hasNext() and next() methods

Processing during traversal

- ▶ May register event listeners to traversal steps
 - ▶ void **addTraversalListener**(TraversalListener<V,E> l)
- ▶ TraversalListeners may react to:
 - ▶ Edge traversed
 - ▶ Vertex traversed
 - ▶ Vertex finished
 - ▶ Connected component started
 - ▶ Connected component finished

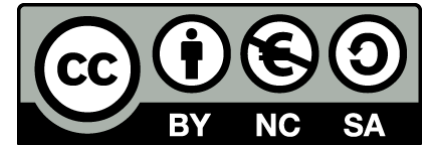
Types of traversal iterators






- ▶ **BreadthFirstIterator**
- ▶ **DepthFirstIterator**
- ▶ **ClosestFirstIterator**
 - ▶ The metric for *closest* here is the path length from a start vertex. `Graph.getEdgeWeight(Edge)` is summed to calculate path length. Optionally, path length may be bounded by a finite radius.
- ▶ **TopologicalOrderIterator**
 - ▶ A topological sort is a permutation p of the vertices of a graph such that an edge $\{i,j\}$ implies that i appears before j in p . Only directed acyclic graphs can be topologically sorted.

Resources

- ▶ JGraphT Library: <http://jgrapht.org/>

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