POLITECNICO DI TORINO

e-Lite

Graph Visits

Tecniche di Programmazione – A.A. 2020/2021

Summary

- Graph visits
- Visits in JGraphT

Graph visits

Representing and visiting graphs

Visit Algorithms

- Visit =
  - Systematic exploration of a graph
  - Starting from a ‘source’ vertex
  - Reaching all reachable vertices
- Main strategies
  - Breadth-first visit (“in ampiezza”)
  - Depth-first visit (“in profondità”)

Breadth-First Visit

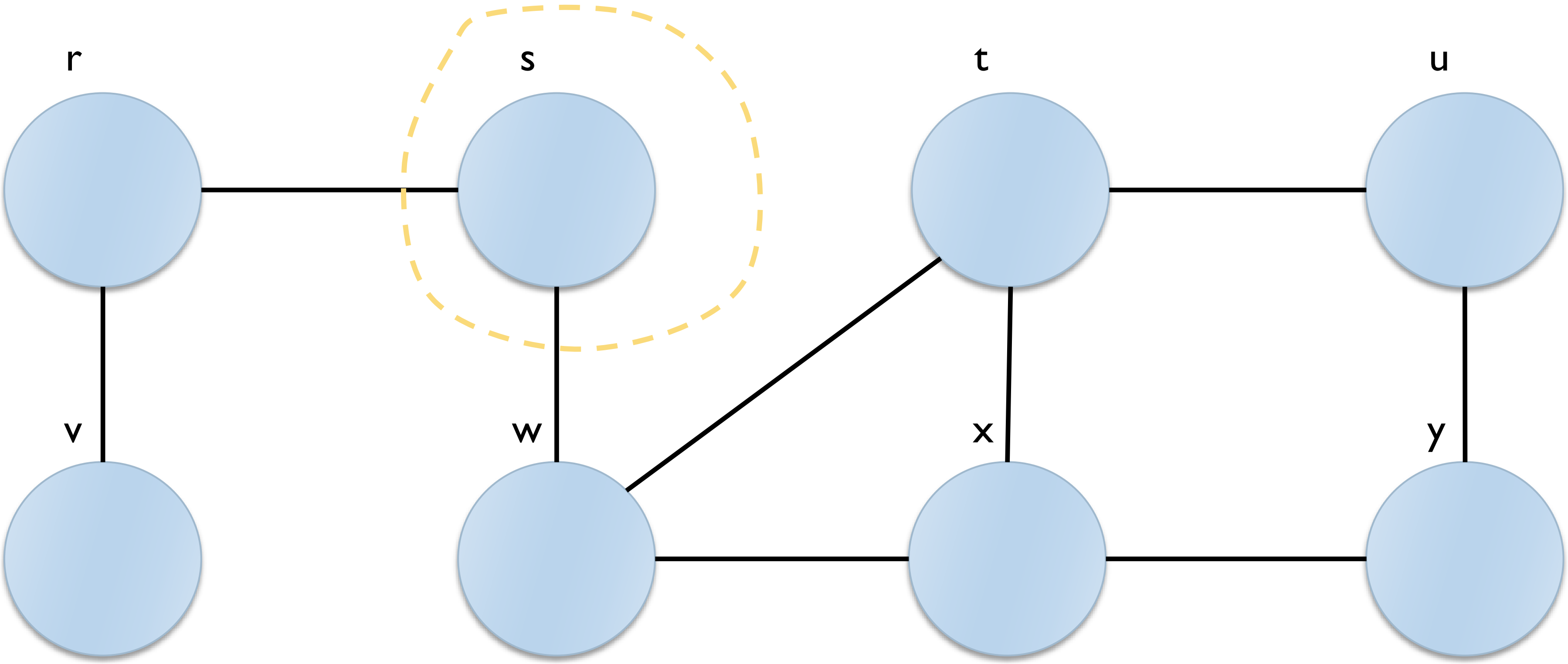
- Also called Breadth-first search (BFV or BFS)
- All reachable vertices are visited "by levels"
  - L – level of the visit
  - S_L – set of vertices in level L
  - $L=0$, $S_0=\{ v_{source} \}$
  - Repeat while S_L is not empty:
    - S_{L+1} = set of all vertices:
      - not visited yet, and
      - adjacent to at least one vertex in S_L
    - $L=L+1$

Example

Source = s

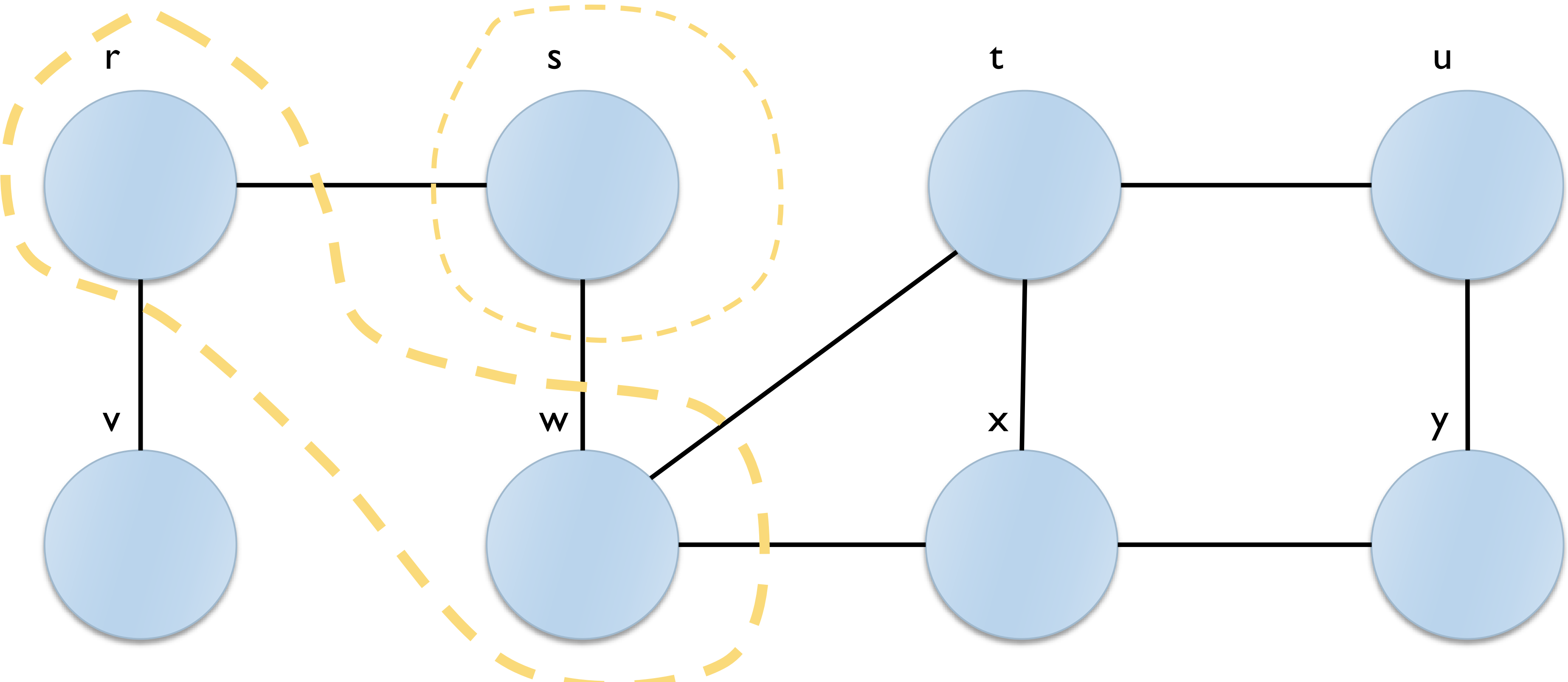
L = 0

$S_0 = \{s\}$



Example

$L = 1$
$S_0 = \{s\}$
$S_1 = \{r, w\}$

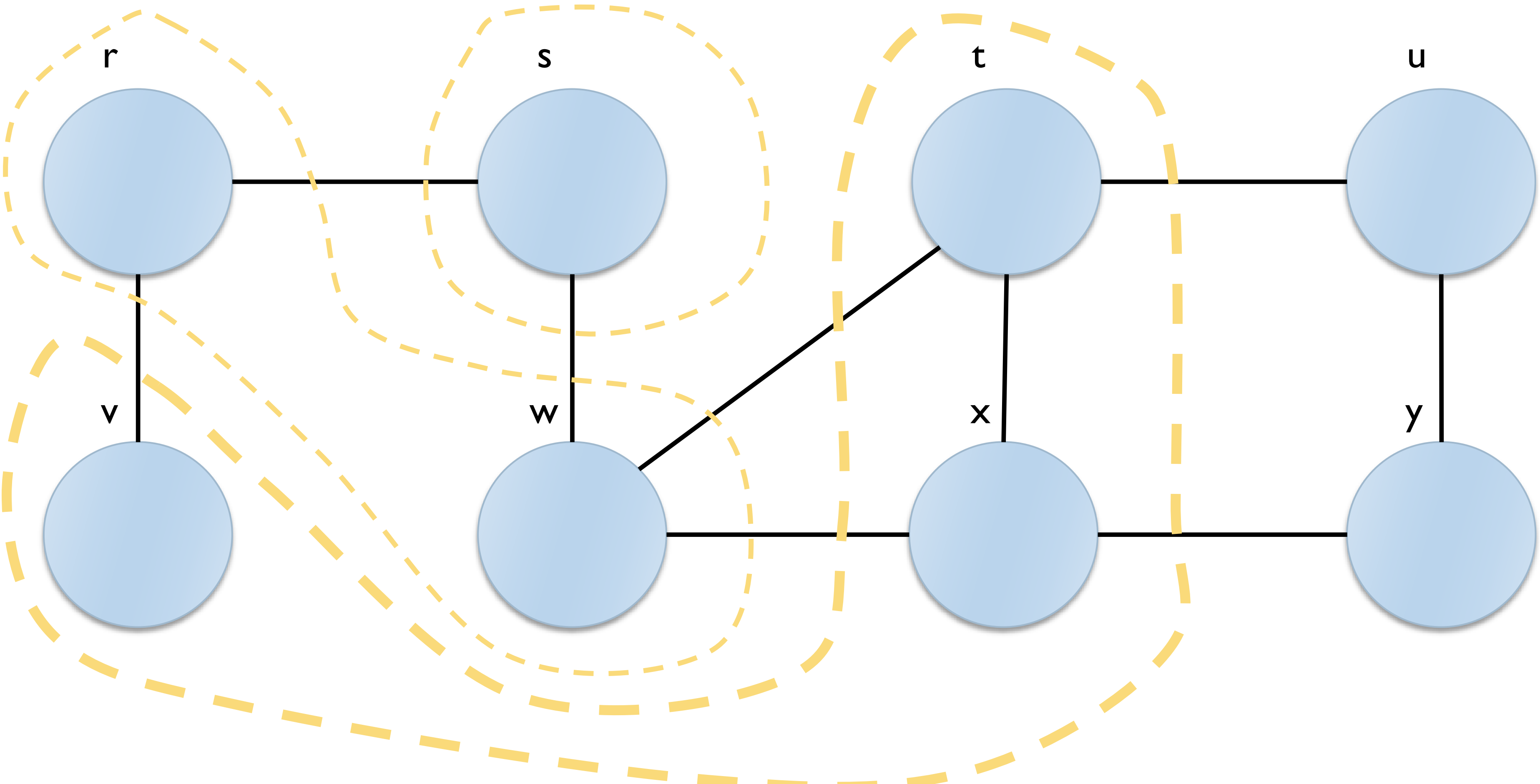


Example

$L = 2$

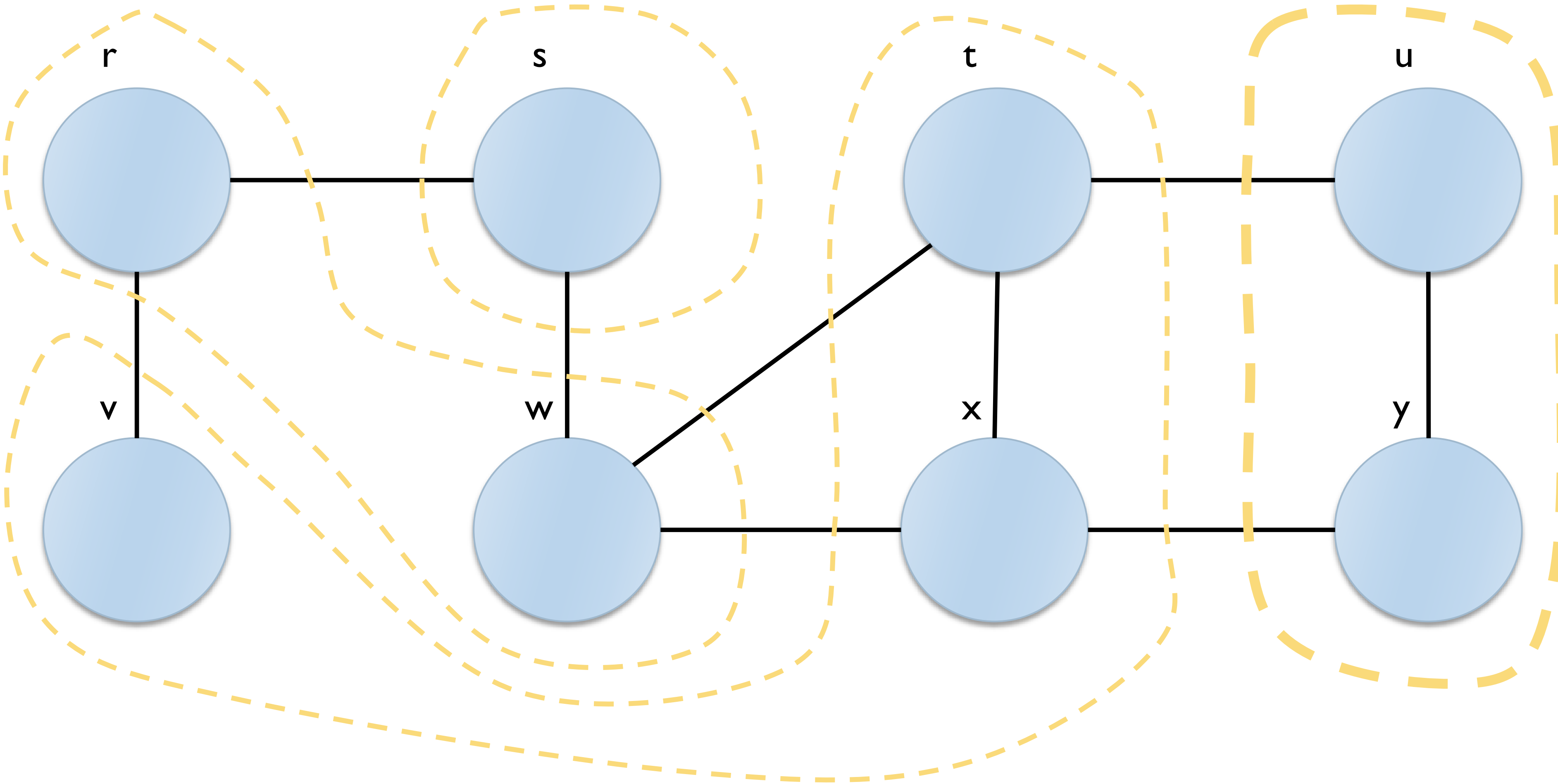
$S_1 = \{r, w\}$

$S_2 = \{v, t, x\}$



Example

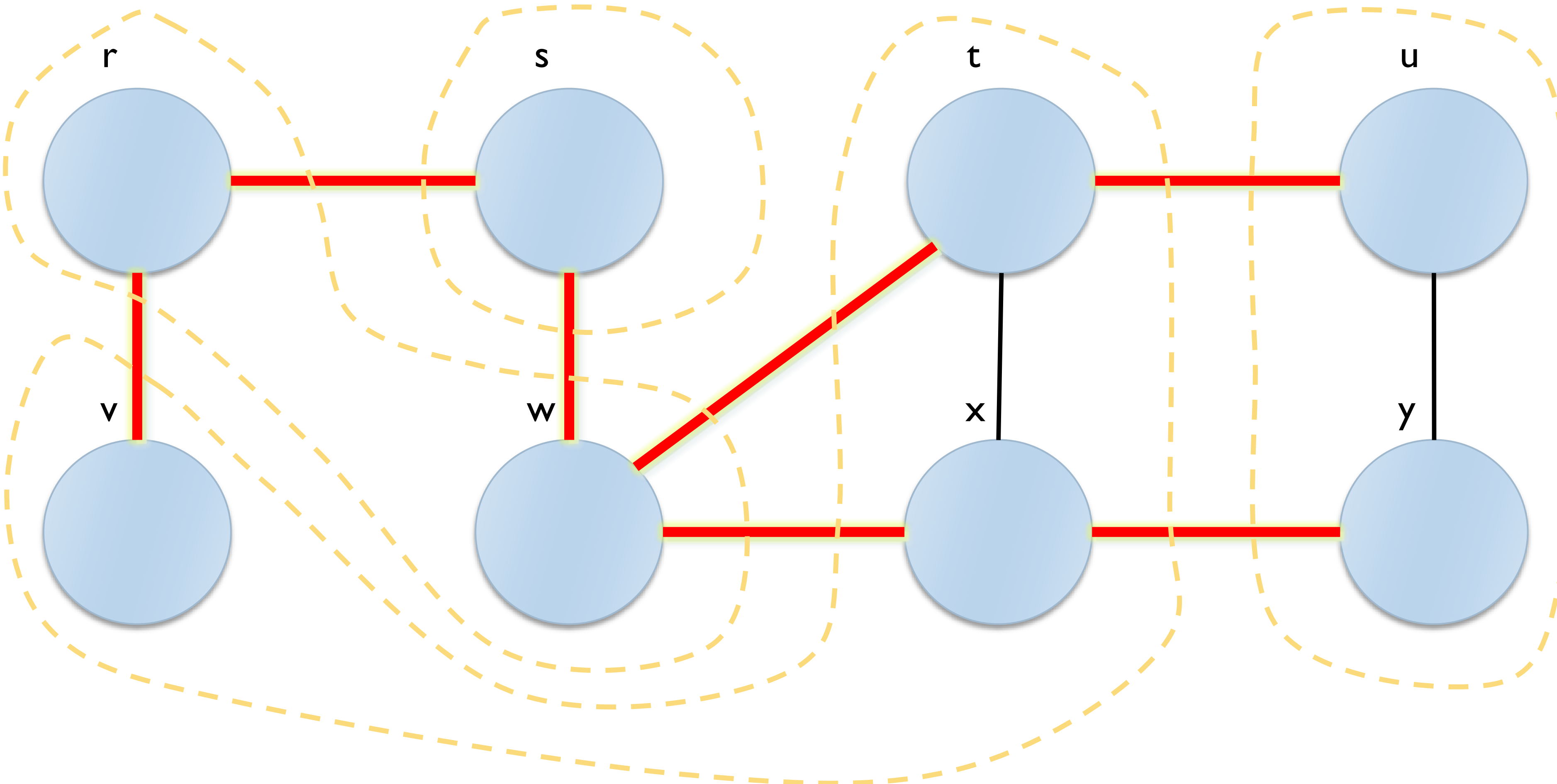
L = 3
$S_2 = \{v, t, x\}$
$S_3 = \{u, y\}$



BFS Tree

- The result of a BFV identifies a "visit tree" in the graph:
  - The tree root is the source vertex
  - Tree nodes are all graph vertices
    - (in the same connected component of the source)
  - Tree are a subset of graph edges
    - Those edges that have been used to "discover" new vertices.

BFS Tree



Minimum (shortest) paths

- Shortest path: the minumum number of edges on any path between two vertices
- The BFS procedure computes all minimum paths for all vertices, starting from the source vertex
- NB: unweighted graph : path length = number of edges

Depth First Visit

- Also called Depth-first search (DFV or DFS)
- Opposite approach to BFS
- At every step, visit one (yet unvisited) vertex, adjacent to the last visited one
- If no such vertex exist, go back one step to the previously visited vertex
- Lends itself to recursive implementation
  - Similar to tree visit procedures

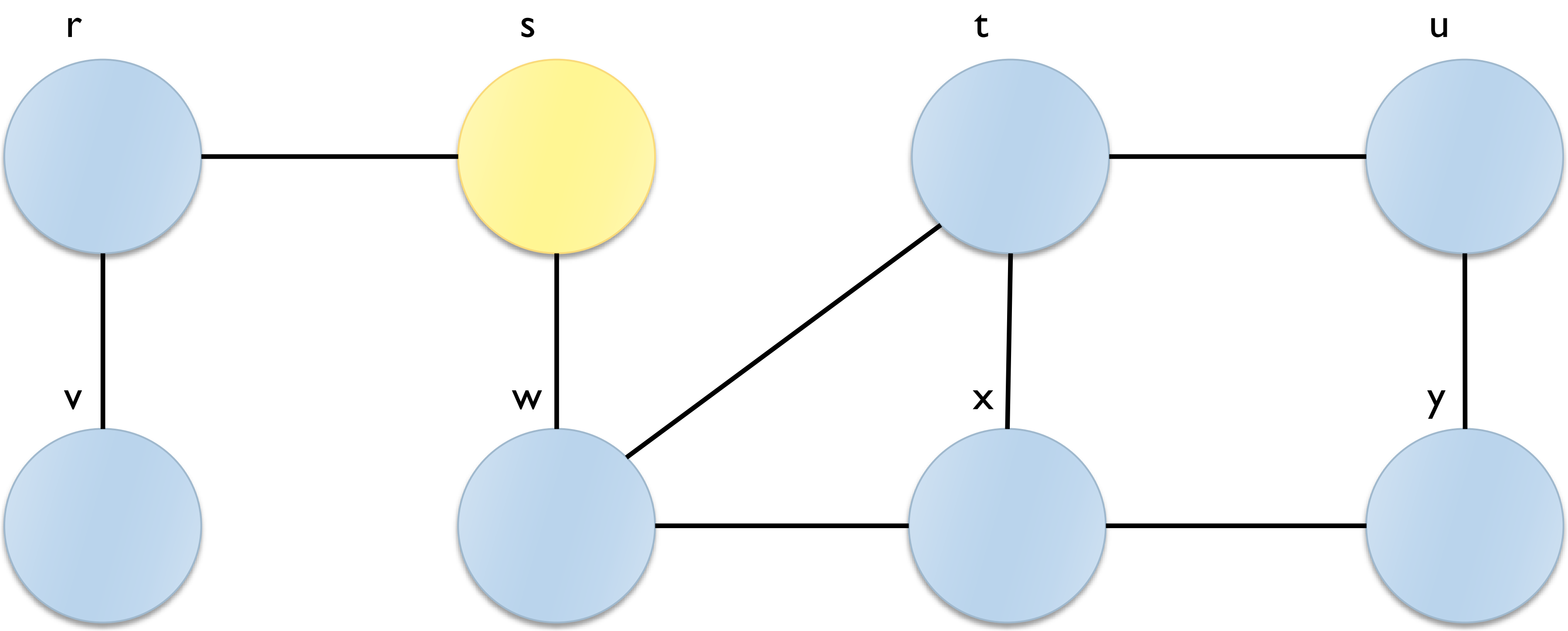
DFS Algorithm

- DFS(Vertex v)
  - For all (w : adjacent_to(v))
    - If(not visited (w))
      - Visit (w)
      - DFS(w)

- Start with: DFS(source)

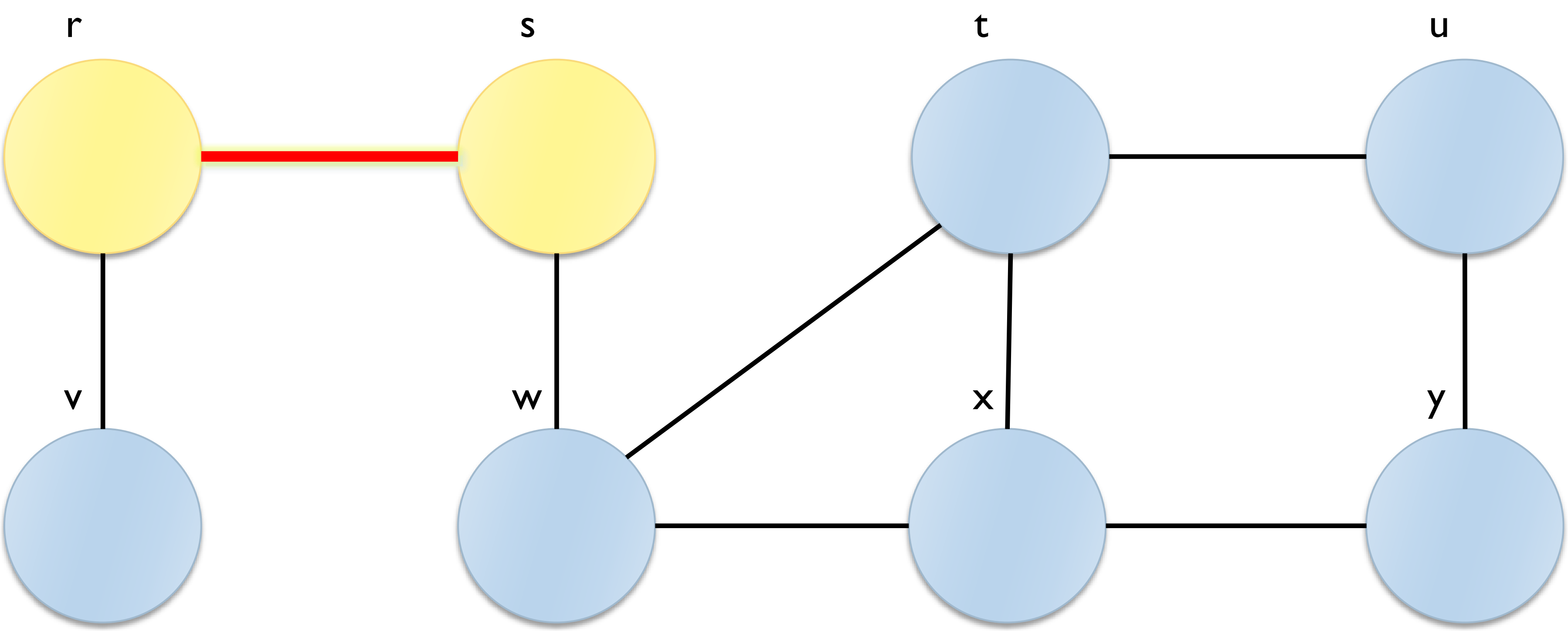
Example

Source = s



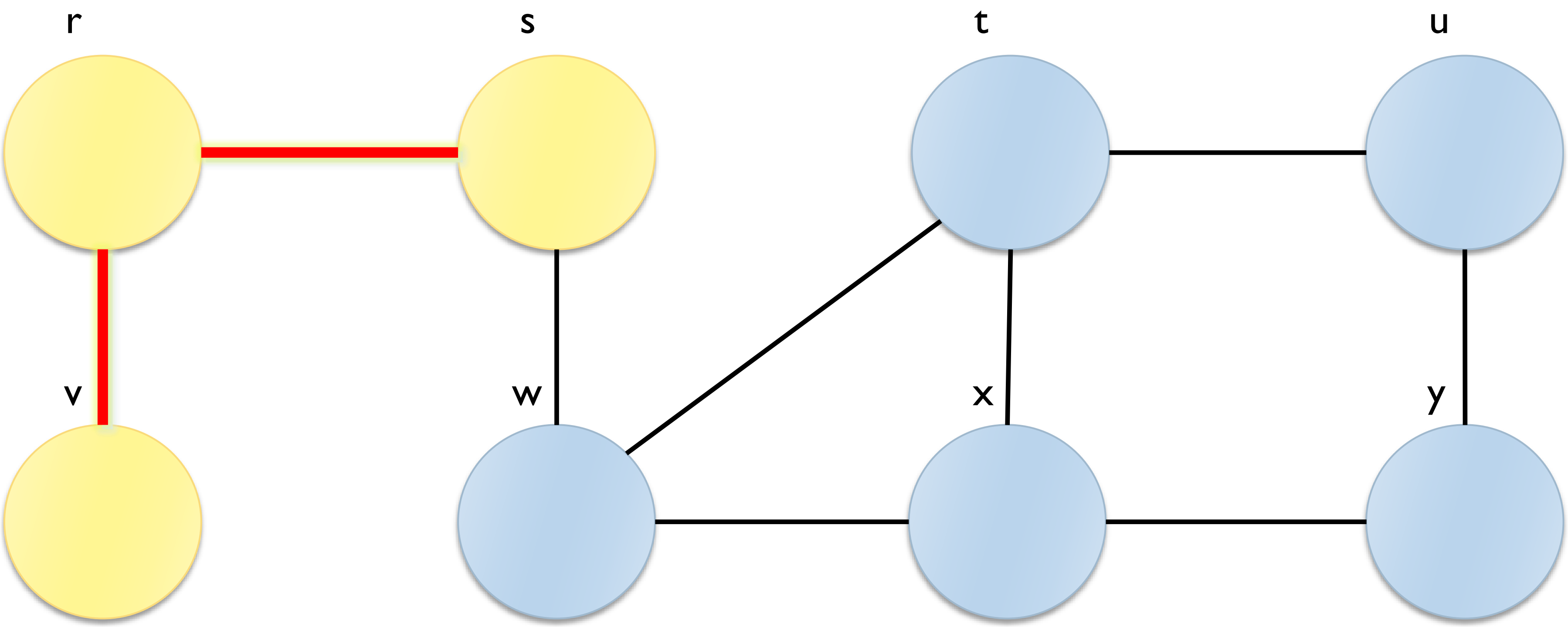
Example

Source = s
Visit r



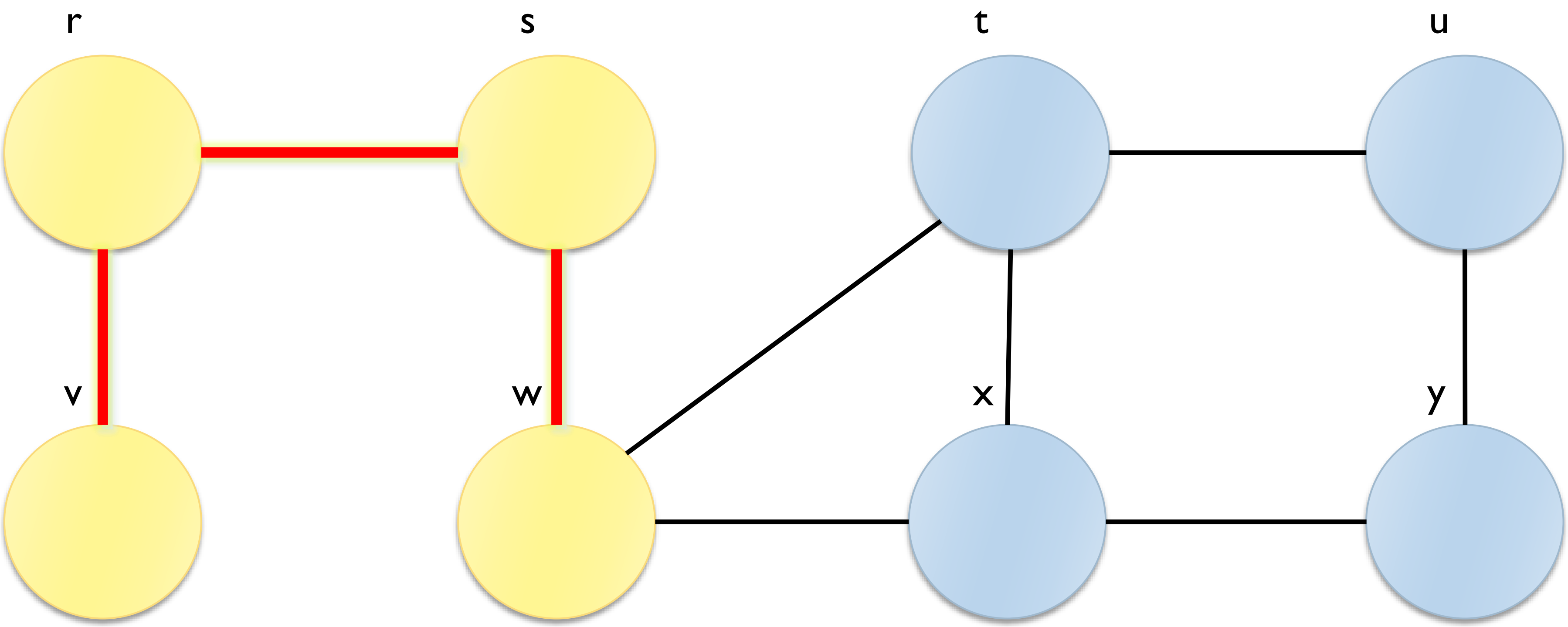
Example

Source = s
Visit r
Visit v



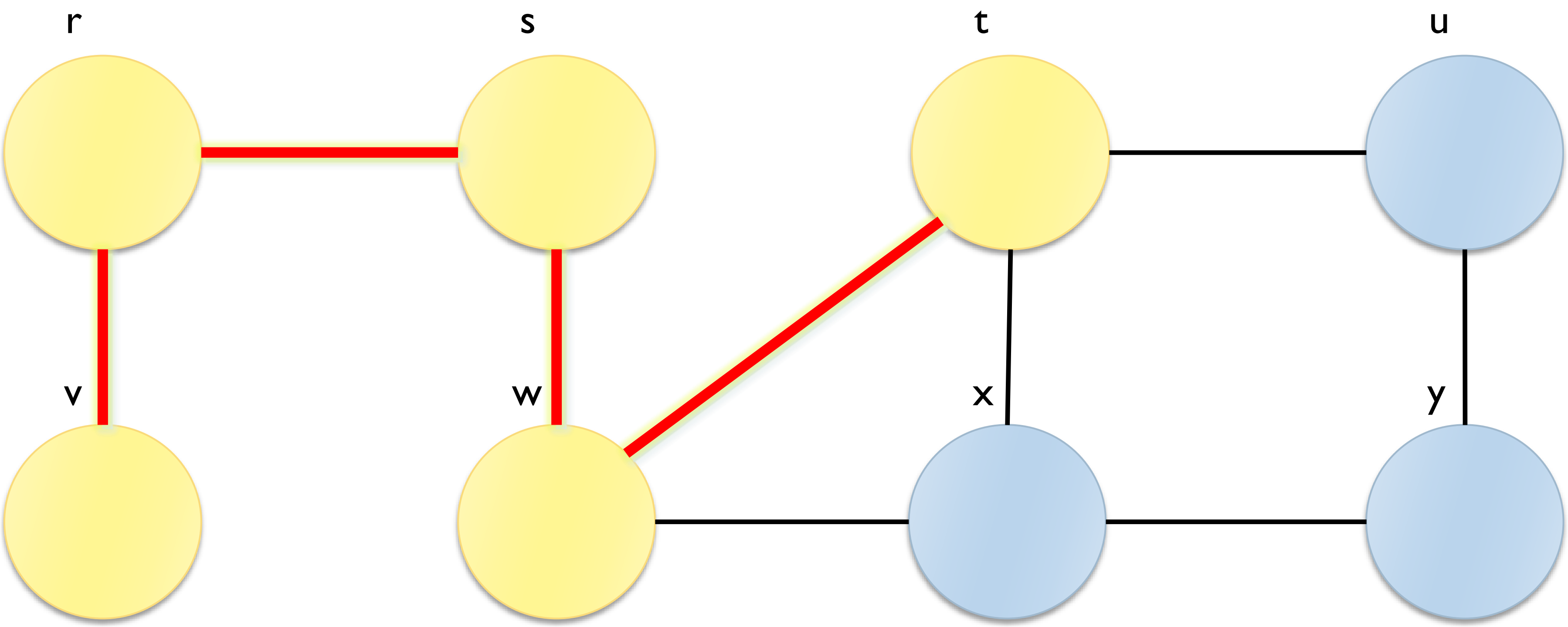
Example

Source = s
Back to r
Back to s
Visit w



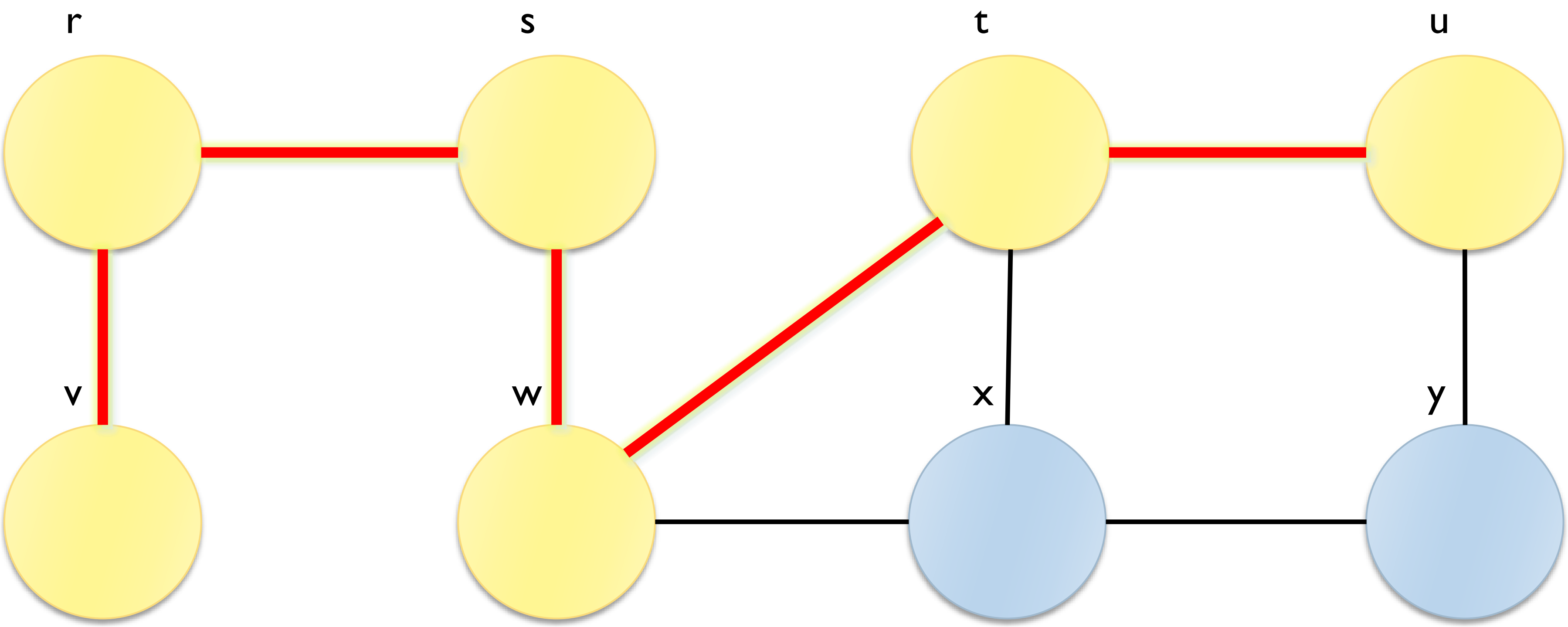
Example

Source = s
Visit w
Visit t



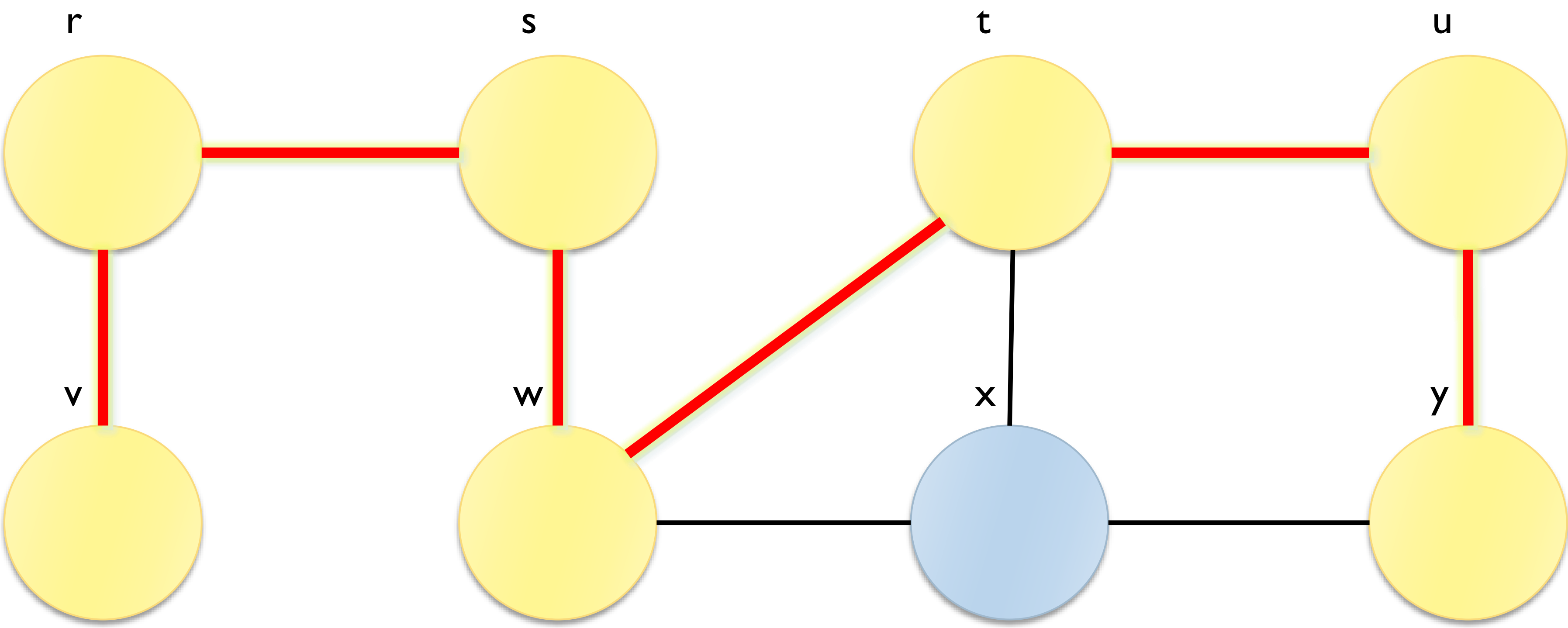
Example

Source = s
Visit w
Visit t
Visit u



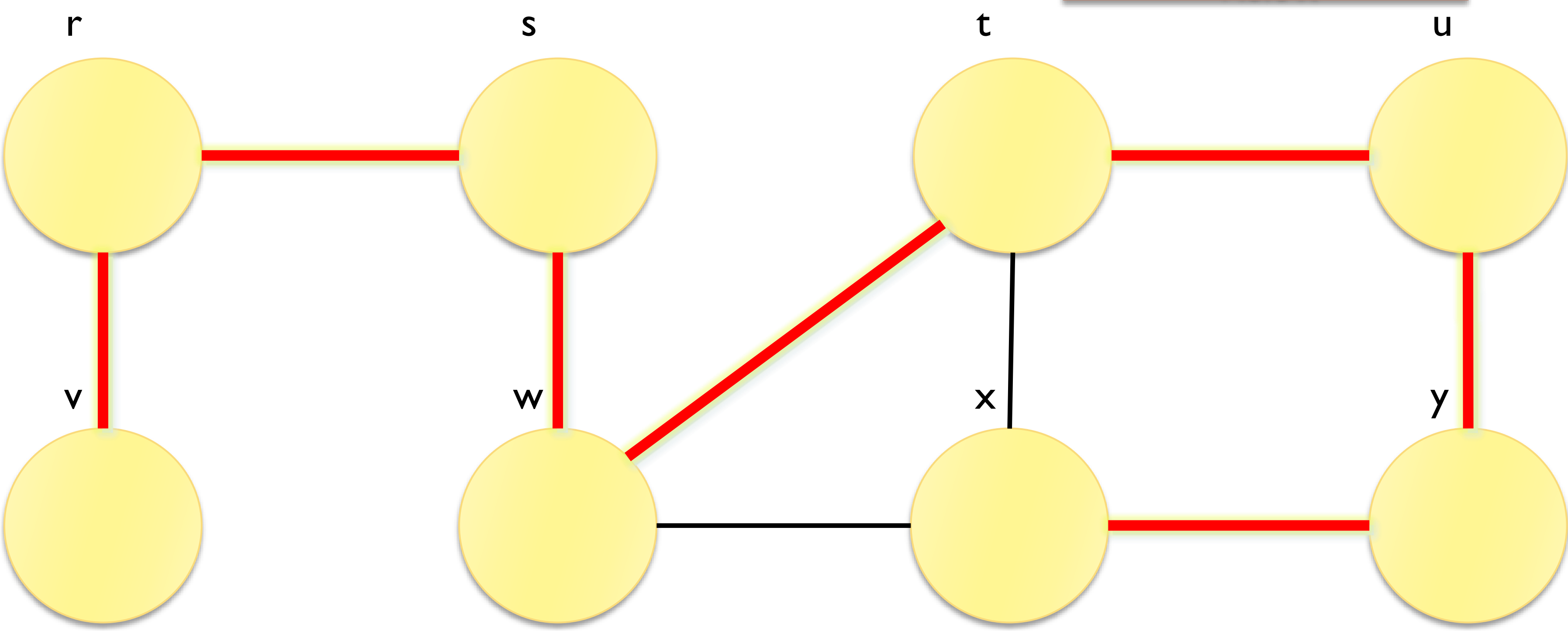
Example

Source = s
Visit w
Visit t
Visit u
Visit y

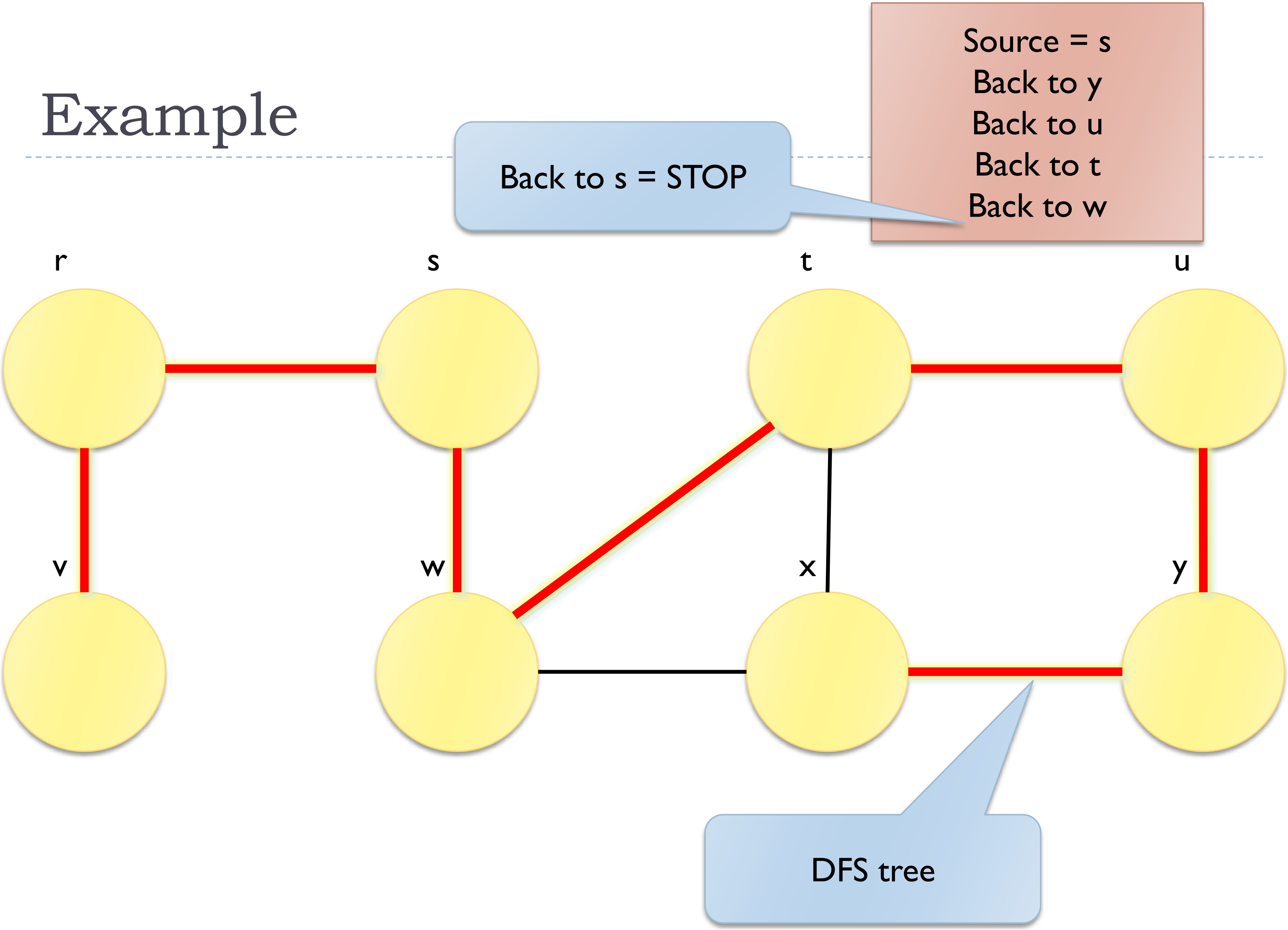


Example

Source = s
Visit w
Visit t
Visit u
Visit y
Visit x

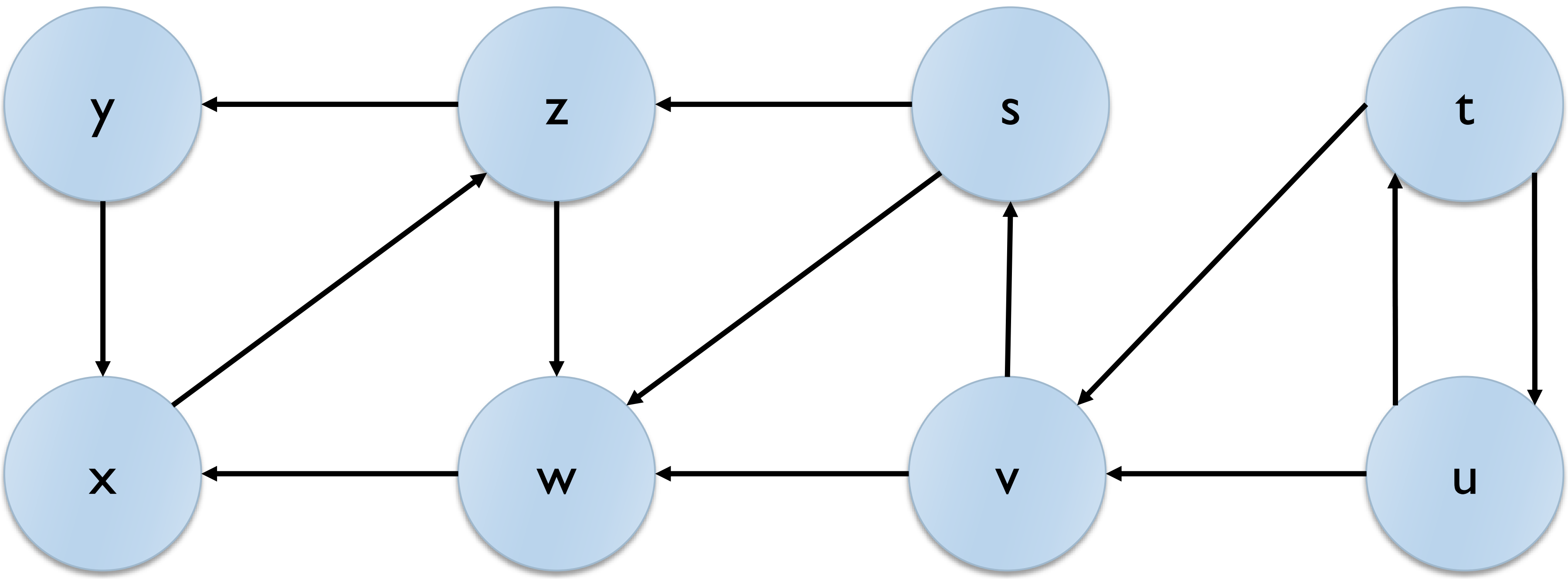


Example


Source = s
Back to y
Back to u
Back to t
Back to w
Back to s = STOP
r
s
t
u
v
w
x
y
DFS tree

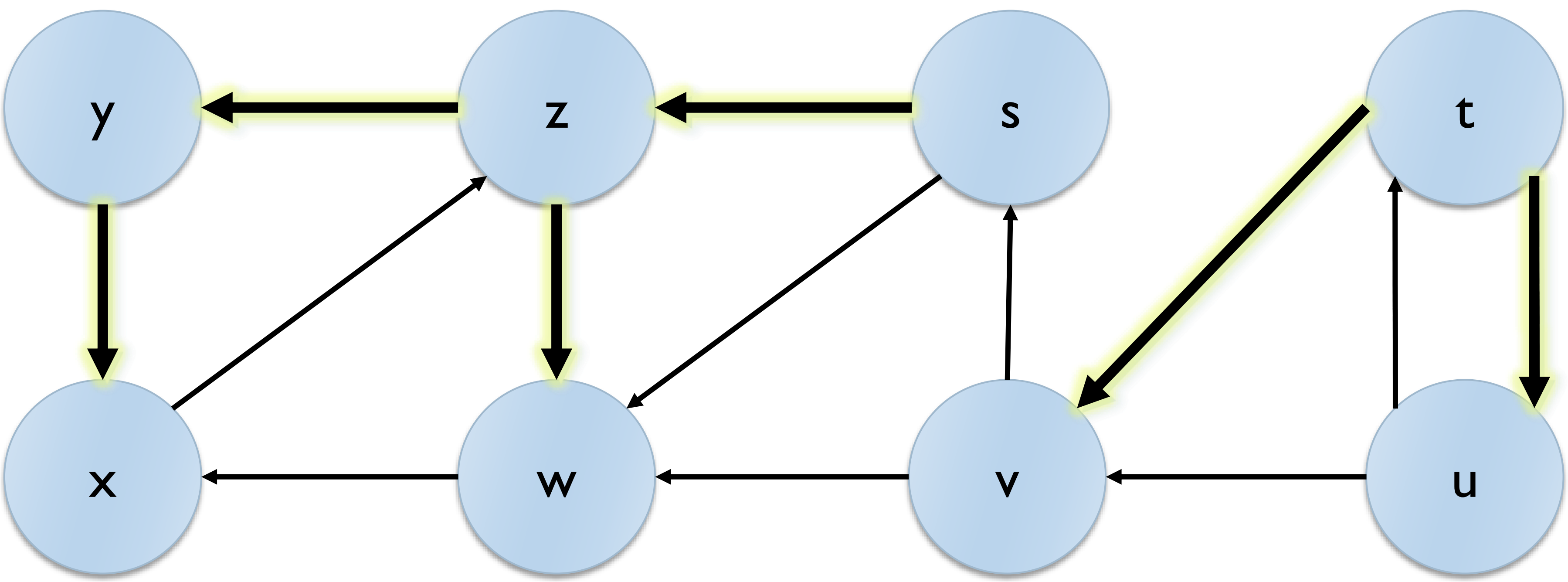
Example

Directed graph



Example

DFS visit
(sources: s, t)



Complexity

- Visits have linear complexity in the graph size
  - BFS : $O(V+E)$
  - DFS : $\Theta(V+E)$
- N.B. for dense graphs, $E = O(V^2)$

Resources

- Maths Encyclopedia: http://mathworld.wolfram.com/
- Basic Graph Theory with Applications to Economics
  http://www.isid.ac.in/~dmishra/mpdoc/lecgraph.pdf
- Application of Graph Theory in real world

http://prezi.com/tseh1wvpves-/application-of-graph-theory-in-real-world/

Resources

- Open Data Structures (in Java), Pat Morin, http://opendatastructures.org/
- Algorithms Course Materials, Jeff Erickson, http://www.cs.uiuc.edu/~jeffe/teaching/algorithms/
- Graphbook - A book on algorithmic graph theory, David Joyner, Minh Van Nguyen, and David Phillips, https://code.google.com/p/graphbook/

Visits in JGraphT

Representing and visiting graphs

JGraphT and visits

- Visits are called “traversals”
- Implemented through **Iterator** classes
- Package **org.jgrapht.traverse**

Graph traversal classes

Package org.jgrapht.traverse

Graph traversal means.

Interface Summary

Interface	Description
GraphIterator<V,E>	A graph iterator.

Class Summary

Class	Description
AbstractGraphIterator<V,E>	An empty implementation of a graph iterator to minimize the effort required to implement graph iterators.
BreadthFirstIterator<V,E>	A breadth-first iterator for a directed or undirected graph.
BreadthFirstIterator.SearchNodeData<E>	Data kept for discovered vertices.
ClosestFirstIterator<V,E>	A closest-first iterator for a directed or undirected graph.
CrossComponentIterator<V,E,D>	Provides a cross-connected-component traversal functionality for iterator subclasses.
DegeneracyOrderingIterator<V,E>	A degeneracy ordering iterator.
DepthFirstIterator<V,E>	A depth-first iterator for a directed or undirected graph.
LexBreadthFirstIterator<V,E>	A lexicographical breadth-first iterator for an undirected graph.
MaximumCardinalityIterator<V,E>	A maximum cardinality search iterator for an undirected graph.
RandomWalkIterator<V,E>	Deprecated. *Use `RandomWalkVertexIterator` instead.*
RandomWalkVertexIterator<V,E>	A random walk iterator.
TopologicalOrderIterator<V,E>	A topological ordering iterator for a directed acyclic graph.

https://jgrapht.org/javadoc/org.jgrapht.core/org/jgrapht/traverse/package-summary.html

Graph iterators

- May be initialized with a start vertex, a set of start vertices, or no vertices (the algorithm chooses)
- Usual `hasNext()` and `next()` methods
- Every time you call `next()` a new vertex V is returned
- When `hasNext()==false`, no more reachable vertices exist

Processing during traversal

- May register event listeners to traversal steps
  - void **addTraversalListener**(TraversalListener<V,E> l)
- TraversalListeners may react to:
  - Edge traversed
  - Vertex traversed
  - Vertex finished
  - Connected component started
  - Connected component finished

Types of traversal iterators

- BreadthFirstIterator
- DepthFirstIterator
- ClosestFirstIterator
  - The metric for *closest* here is the path length from a start vertex. Graph.getEdgeWeight(Edge) is summed to calculate path length. Optionally, path length may be bounded by a finite radius.
- TopologicalOrderIterator
  - A topological sort is a permutation *p* of the vertices of a graph such that an edge {*i*,*j*} implies that *i* appears before *j* in *p*. Only directed acyclic graphs can be topologically sorted.

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